Teaching for Robust Understanding with Lesson Study

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Abstract

This chapter describes the synthesis of two powerful approaches to professional development, based on the Teaching for Robust Understanding (TRU) framework and Lesson Study. The synthesis is known as TRU-Lesson Study.

The TRU framework identifies five essential dimensions of classroom practice: (1) the Content, (2) Cognitive Demand, (3) Equitable Access, (4) Agency, Ownership, and Identity, and (5) Formative Assessment. When classroom practices engage students meaningfully along these five dimensions, students become knowledgeable and resourceful thinkers and problem solvers. In TRU-based professional development, groups of teachers negotiate their visions of teaching and learning collaboratively by reflecting on artifacts of practice using the TRU framework.

In math-focused Lesson Study (LS), teachers work together to design, teach, and reflect on a lesson that focuses on key mathematical issues and students' engagement with them. Widespread in Japan, Lesson Study is a powerful mechanism for building and sharing understandings of mathematics content, teaching and learning.

Unlike in Japan, teachers in the US typically have little collective time to reflect on teaching practice; opportunities to negotiate shared understandings of what matters for learning are rare. The tools developed for TRU professional development address these challenges by providing explicit support for community building around the five key dimensions of the TRU framework.

TRU-Lesson Study, like Lesson Study, profits from teachers' concerted attention to lesson design and reflection on the hypotheses reflected in the design. Like TRU professional development, it supports teachers to work together explicitly on key dimensions of classroom practice. This paper describes TRU-Lesson Study and provides descriptions of how it plays out in practice.

1. Background and context

1.1 Introduction and Overview

As this volume indicates, Lesson Study (LS) is a worldwide mechanism for professional development. In mathematics, Lesson Study's power comes in large part from teachers' collaborative efforts to understand the richness and complexity of mathematical ideas and practices and to support students in engaging with them. Teachers' collective negotiations of the problems of practice that arise when engaging in ambitious pedagogy – designing and reflecting on lessons that focus on rich content and student learning of it – provide an ongoing mechanism to share and develop pedagogical understandings.

The Teaching for Robust Understanding (TRU) framework (see Schoenfeld, 2013, 2014, 2015, in press; Schoenfeld & the Teaching for Robust Understanding Project, 2016) describe what matters for equitable and robust learning in mathematics classrooms. As elaborated in Section 1.3, the TRU framework identifies five key dimensions of learning environments: (1) The Content (in this case the Mathematics); (2) Cognitive Demand; (3) Equitable Access; (4) Agency, Ownership and Identity; and (5) Formative Assessment. The first dimension focuses on the richness and complexity of mathematical ideas and practices essential for powerful mathematics learning. The remaining four dimensions focus on students' experiences of the mathematics and the impacts of those experiences.

TRU professional development (PD) engages communities of mathematics teachers in ongoing inquiry cycles of reflection and practice grounded in the TRU framework. In regularly scheduled meetings, teachers are supported in sharing artifacts from their classrooms that capture evidence of focal problems of practice and use the TRU framework to guide their reflections. Their inquiry continues between meetings as teachers try new strategies, visit each other's classrooms, and/or collect classroom artifacts to bring to the next meeting to further their investigations. Rather than specifying any particular approach to teaching, the TRU framework focuses teachers' attention on attending explicitly to aspects of practice that are essential for supporting students' collaborative engagement with rich mathematical ideas and practices.

There is a strong philosophical affinity between TRU and LS. Both attend explicitly to how students experience mathematics instruction; both see discourse within TLCs as a venue for teachers' ongoing professional growth; and both value teacher professionalism. In Japan, LS is well established, as are teacher collaboration, peer support for collective growth and teacher professionalism. In the US, collaborative support for teacher growth is rare (Horn, Garner, Kane, & Brasel, 2017; Little, 2002). Lortie (1975) characterized school buildings as "egg crates," in which each teacher works in his or her room entirely isolated from colleagues. Today, this is still often the case. TLCs in the US struggle to develop the trust and depth of conversation necessary for inquiring into complex problems of practice in ways that could lead to learning (Horn & Little, 2010; Horn, Garner, Kane, & Brasel, 2017) a significant obstacle to the implementation of Lesson Study. In addition, teachers in the US are not necessarily as well prepared as Japanese teachers to focus in detail on student thinking. TRU-LS is designed to help build teacher communities and to focus on aspects of student thinking in ways that allow the practices of Lesson Study to take hold, enriching what teachers can learn from focused reflection on designing instruction and student engagement with important mathematical ideas.

1.2 Lesson Study model[1]

Lesson Study has established a foothold in the US over the past 15 years (Akiba, Ramp, & Wilkinson, 2014; Hill, 2011). Not surprisingly, there is a range of LS adaptations to the US context. TRU-LS draws on the theoretical model of Lesson Study proposed by Catherine Lewis and colleagues, aimed at adapting Lesson Study to the US context (Lewis, Perry & Hurd, 2009; Lewis, Perry & Murata, 2006; Perry & Lewis, 2009; Takahashi & McDougal, 2016; Watanabe, Takahashi & Yoshida, 2008). The core aspects of one lesson study cycle are shown in Figure 1.

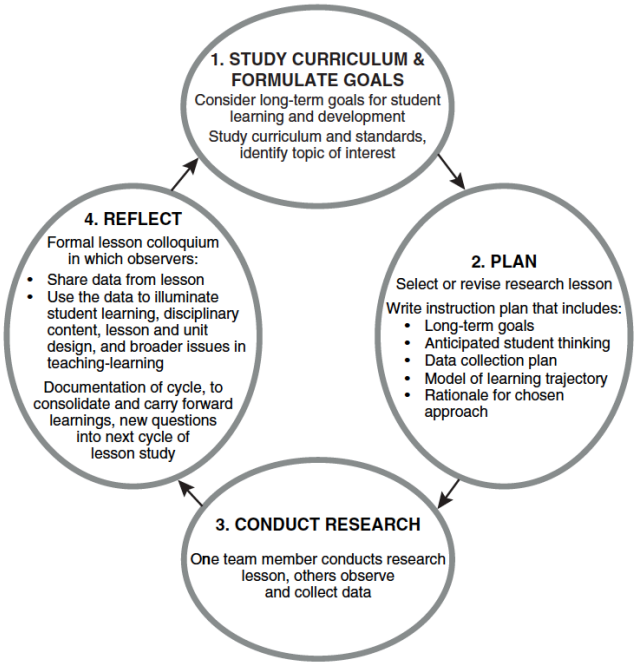

Figure 1. Lesson Study cycle (From Lewis, Perry, & Murata, 2006, p. 4)

In one LS cycle, a teacher learning community (TLC) moves together through the four-phase process in Figure 1. They begin with *kyouzai kenkyuu* or "*study* of materials for teaching." In this phase the TLC decides on a research theme and theory of action and selects a mathematical topic to focus on. It conducts background research, looking at what is known about student understanding of the topic and at relevant instructional materials (possibly including exemplary lessons). With this refined understanding, they *plan* a lesson, identifying precise issues that members of the team want to focus on when the lesson is taught. One or more teachers then *conduct* the research lesson, while the remaining teachers and invited educators observe, gathering evidence of student thinking. Participants then *reflect* on how the lesson played out and discuss their observations with an outside expert. They then move on to their next LS cycle.

In brief, the theory of action underlying LS is that such repeated inquiry cycles support the TLC's sense of community and, in an organic way, deepen and modify teachers' beliefs and understandings. This leads to changes in their pedagogical practices and deeper student learning.

[1] Our description is terse, given that this entire volume is devoted to Lesson Study.

1.3 An introduction to the Teaching for Robust Understanding framework and PD structure

The TRU framework provides a theory of powerful learning environments (Schoenfeld, 2015). It has been used as the multi-disciplinary theory of action spanning all content areas in cities like San Francisco (San Francisco Unified School District, 2016). Tools exist for supporting classroom practices in all disciplines (see, e.g., Baldinger, Louie, and the Algebra Teaching Study and Mathematics Assessment Project, 2016; Schoenfeld & the Teaching for Robust Understanding Project, 2016). In this paper, we focus on the mathematics version of TRU, TRUmath.

1.3.1 The Teaching for Robust Understanding framework

The five dimensions of the TRUmath are given in Figure 2.

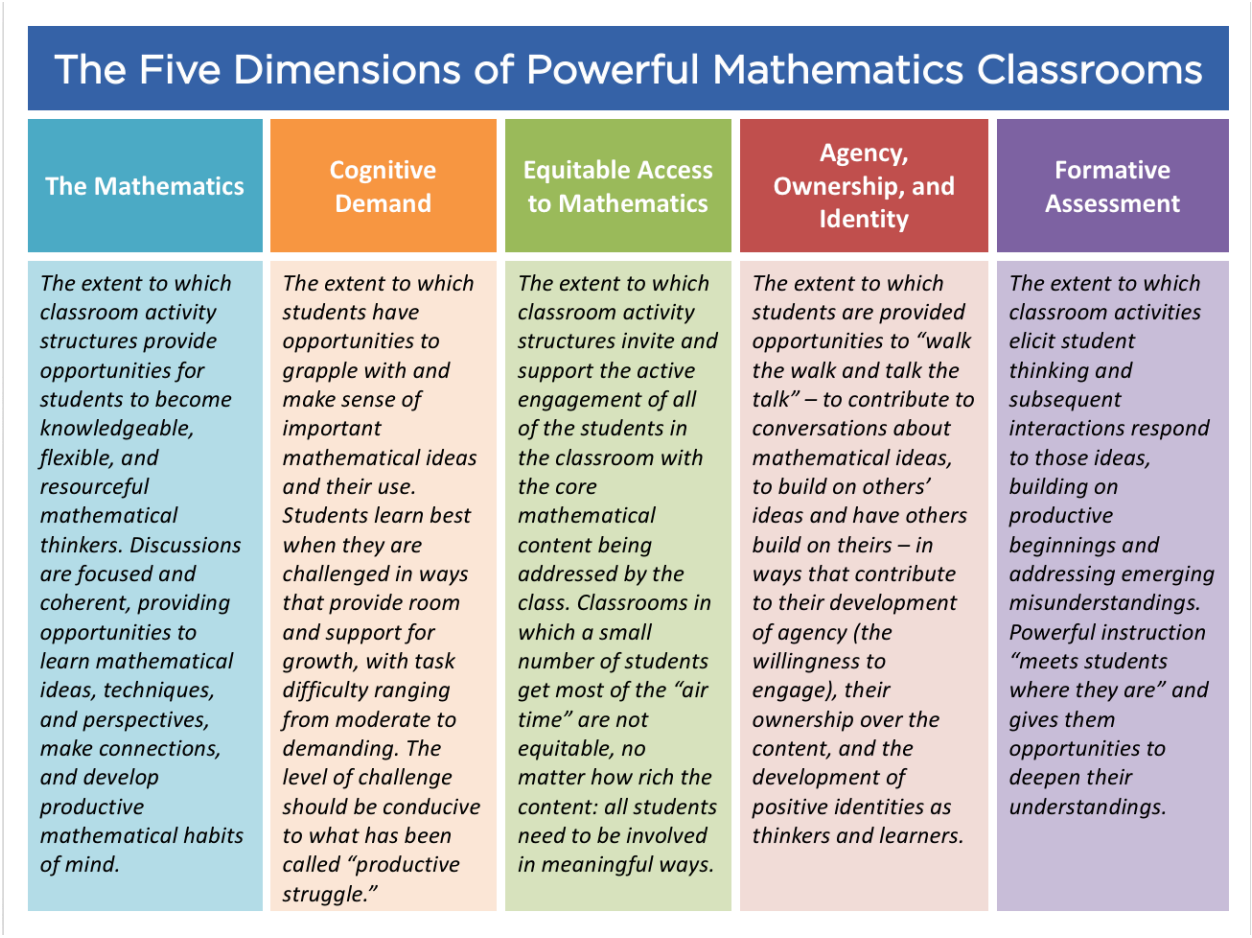

Figure 2. The five dimensions of robust mathematics classrooms –
the Teaching for Robust Understanding (TRU) framework for mathematics, TRUmath

As described in detail by Schoenfeld (2013; 2014; 2018) the derivation of the framework included a comprehensive literature search regarding factors affecting student outcomes and the distillation of that list into five equivalence classes chosen in such a way that each is "actionable" – each can be the focus of professional development. Examinations of videotapes of practice and the use of a scoring rubric (Schoenfeld, Floden, & the Algebra Teaching Study and Mathematics Assessment Project, 2014) documented a positive relationship between scores assigned to classroom practices using the TRUmath rubric and student performance on

measures of mathematical proficiency. Students from classrooms that rate increasingly well along the five dimensions are increasingly knowledgeable and resourceful thinkers and problem solvers.

TRU has several features that afford teacher learning, some of which are enumerated below.

1. *The TRU framework offers a perspective and a language for discussing and reflecting on what matters in instruction. TRU highlights key arenas of practice for teachers to inquire into, experiment with, and reflect on collaboratively.*

Teachers in the US have few opportunities to develop shared goals with their colleagues (Grossman & McDonald, 2008). Consequently, teachers and administrators can struggle to find common ground when beginning to talk about important problems of practice. This, in turn, makes it harder for them to work together to build and achieve shared goals.

The perspective and language offered by TRU serve as a mechanism for boundary-crossing (Akkerman & Bakker, 2011; Star & Griesemer, 1989) and provide a means of communication between teachers at multiple grade levels and administrators. The five dimensions of TRU support teachers in inquiring into and reflecting on each other's routine practice. The hard work of coming to understand what the TRU dimensions can look like in classrooms helps teachers, administrators, parents, and other stakeholders build shared understandings and goals. This facilitates collective work on problems of practice.

2. *In the cultural context of the US, TRU involves a fundamental shift in perspective from focusing on what students are <u>doing</u> in classrooms (the activities and/or materials they engage with) to focusing on the ways in which students experience mathematics (as supported by activities and materials).*

Focusing on the ways students experience instruction is a fundamental point of alignment between TRU and LS. A key issue is, "What does math class feel like, from the point of view of the student?" This perspective is represented in Figure 3, drawn from the TRU observation guide (Schoenfeld and the Teaching for Robust Understanding Project, 2016).

Observe the lesson through a student's eyes	
The Mathematics	• What's the big idea in this lesson? • How does it connect to what I already know?
Cognitive Demand	• How long am I given to think, and to make sense of things? • What happens when I get stuck? • Am I invited to explain things, or just give answers?
Equitable Access to Mathematics	• Do I get to participate in meaningful mathematical learning? • Can I hide or be ignored?
Agency, Ownership, and Identity	• Do I get to explain, to present my ideas? Are they built on? • Am I recognized as being capable and able to contribute in meaningful ways?
Formative Assessment	• Do classroom discussions include my thinking? • Does instruction respond to my thinking and help me think more deeply?

Figure 3. Observing a mathematics lesson from the student perspective

It can be challenging for teams of teachers in the US to move their conversations beyond what students will *do* in class, including scheduling activities, organizing curriculum, and naming topics to be taught. Such conversations are necessary, but they rarely reach the conceptual depth that leads to powerful teacher learning (Horn, Garner, Kane & Brasel, 2017). TRU shifts conversations away from activities and materials to what students *perceive and experience*, supporting teachers in exploring goals and orientations towards teaching and learning. Questions such as, "What does it feel like for a student to experience these mathematical ideas for the first time? How does this connect to what students already know? What's confusing, interesting, or useful about this?" do not have easy answers. They address what really matters for students and are worthy of extended, collaborative exploration.

3. *There are many different ways to design a mathematically rich learning environment. Accordingly, the TRU framework does not prescribe specific teaching methods. Professional development aligned with the TRU framework provides mechanisms that help teachers build on their strengths and refine their practices toward providing their students with increasingly rich experiences along each of the five TRU dimensions.*

The TRU framework points to important ways to *problematize* instruction, opening it up for inquiry and reflection, while respecting teacher professionalism and autonomy. Specifically, asking, "How are students experiencing the mathematics along this dimension? How can those experiences be made richer?" allows teachers the freedom to work within their own styles, while building shared understanding of and commitment to a set of goals known to help students become powerful mathematical thinkers. TRU is not laissez-faire: reflecting using the TRU dimensions in a particular context may well reveal that one particular approach in that context is more likely to foster deep student engagement with the content, or provide more equitable access to it, than another approach. But TRU does not tell teachers what to do. It supports and enhances reflection on aspects of instruction that matter.

1.3.2 TRU professional development

TRU professional development takes advantage of the three affordances of the TRU framework described above – explicit foci, shift to the student perspective, and the structured problematizing of teaching methods – to help teachers build communities of inquiry around important problems of practice. TRU-based professional development entails a series of regular inquiry cycles in which teachers negotiate their visions of teaching and learning mathematics collaboratively by reflecting on artifacts of practice and then trying out new approaches with their students. Each stage of this inquiry cycle is supported by the explicit foci and boundary crossing language built into the TRU framework.

Once teams of teachers have been introduced to TRU they begin the inquiry process with collective reflections (top part of Figure 4). The TRU framework facilitates the choice of a problem of practice that will be central to their collaborative inquiry[2]. Having chosen a focal

[2] Some examples of problems of practice and the TRU dimensions that support them are, "How can we support students to feel ownership over the mathematics and discuss each other's mathematical ideas and strategies?" (TRU Dimension 4: Agency, Ownership and Identity) or

problem of practice, the teachers collaboratively identify relevant activities or techniques to try in their classrooms. The framework orients them to what they might try, but they are free to experiment with a variety of methods.

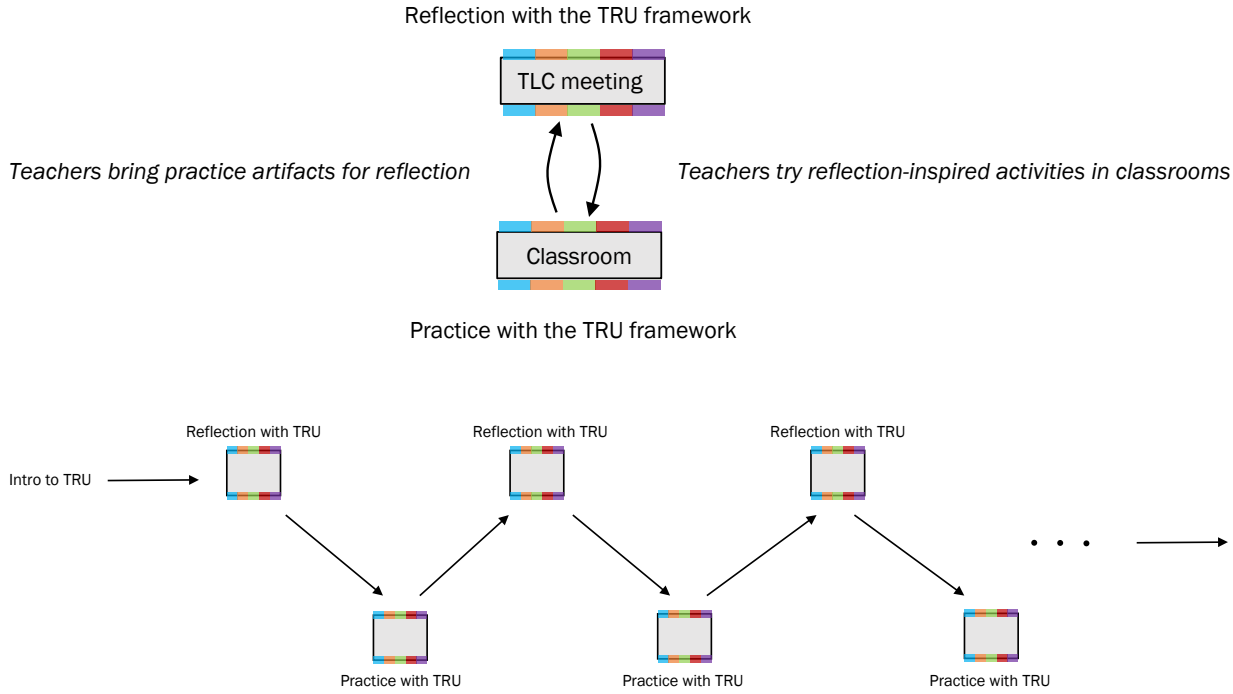

Figure 4: The TRU inquiry cycle

At this point, their inquiry shifts into their classroom practice. Teachers enact their interventions, collecting artifacts that will ground their subsequent collaborative reflections. Those artifacts might include replays or otherwise rich descriptions of what took place (see, e.g., Horn, 2005), student work (Kazemi & Franke, 2004), or video (Sherin, Linsenmeier & van Es, 2009).

In the next TLC meeting, the teachers report back to the group and reflect collectively on what happened. They use TRU tools such as the *TRU Conversation Guide* (Baldinger, Louie, and the Algebra Teaching Study and Mathematics Assessment Project, 2016) and the *TRU Observation Guide* (Schoenfeld, Floden and the Algebra Teaching Study and Mathematics Assessment Project, 2016) to focus their reflections on students' experiences[3]. Through reflection, teachers negotiate interpretations of what happened and future plans. They then plan another intervention to refine their understanding of their chosen problem of practice, and the cycle again returns to the classroom.

Through the repeated cycles of reflection and practice shown in Figure 4, TRU PD aims to develop teachers' capacity to collaborate on enriching students' experiences of mathematics

"How can we support students to struggle productively?" (TRU Dimension 2: Cognitive Demand).

[3] See http://truframework.org for some of the tools developed to support teachers' planning, observation, and reflection.

and mathematics instruction by building routines of inquiring into typical practice. Working collaboratively with multiple TRU inquiry cycles both helps to build a sense of teacher community and supports teachers in structured collective reflection on their attempts to create increasingly rich mathematical learning environments.

1.4. The synthesis of TRU and LS

The goal of synthesizing TRU and LS was to leverage the strengths of both TRU PD and LS, employing TRU PD activities and the TRU framework to address the challenges of shifting LS from the Japanese to the US context. Research has highlighted a number of these challenges while concurrently enumerating critical features deemed necessary for Lesson Study to be effective (Fernandez, Cannon & Chokshi, 2003; Perry, & Lewis, 2009; Takahashi & McDougal, 2016; Watanabe, Takahashi & Yoshida, 2008). One such critical feature is that a significant amount of time needs to be spent on *kyouzai kenkyuu*[4]. As discussed below, this feature is addressed with TRU's integration into LS and its focus on building expertise about what matters in mathematics teaching and learning. Specifically, TRU-LS does so by providing teachers with opportunities for in-depth explorations of student thinking in their own classrooms, while laying the groundwork for and implementing Lesson Study research cycles occurring on a larger time scale. Two main TRU integrations into Lesson Study produce the synthesis and tackle these challenges.

First, as shown in Figure 5, TRU-LS incorporates the TRU framework into each stage of the Lesson Study process. Much as teachers use the TRU dimensions to frame their inquiry into their chosen problem of practice during TRU PD, teacher learning communities engaged in TRU-LS use the TRU framework to frame the different stages of Lesson Study. TRU helps teachers select, study, and refine their research theme and theory of action, and provides explicit focus for *kyouzai kenkyuu*. TRU observation tools help teachers focus their data collection during the research lesson on what students experience. The TRU framework also guides the final commentary during the post-lesson discussion. In the US context, where students' potential experiences of mathematics are not always central to conversations about teaching, the focus that the TRU framework provides is essential. It helps keep the focus on building expertise around student thinking.

Second, the TRU-LS process insures that all members of the department or TLC are engaged with all aspects of the research lesson before it is taught by folding the TRU PD inquiry cycles into the larger Lesson Study cycle. As seen in Figure 5, the TRU inquiry cycles form the backbone of the study phase of Lesson Study. The research theme and preliminary theory of action for the research lesson are developed in the TLC. All TLC members (including the planning team) then engage further with the research theme and theory of action through repeated cycles of reflection and practice. This ongoing work allows for collective refinement of

[4] Focused attention on the mathematics and to student thinking is essential in the US context. Curricula in the US vary widely and are very uneven in quality, so teachers in the US cannot depend on the affordances of curricula that are available in Japan. Moreover, teachers in the US have not, as a rule, had nearly as much opportunity as Japanese teachers to reflect on the impact of classroom materials and practices on student thinking.

the research theme and theory of action. Later, as the planning team works to design the research lesson, it brings the core mathematics of the lesson to the larger community for discussion. That conversation includes discussions of the mathematics itself, what the students are likely to know, anticipated student responses and how mathematical ideas develop throughout the lesson as well as specific design elements. These collective conversations inform the research lesson planning and also raise possibilities for intervention and reflection that the whole department can try out in ongoing inquiry cycles. As shown in Figure 5, the plan, do, and reflect phases of Lesson Study are each one node in the sequence of reflection and practice that make up the inquiry in TRU PD. Note specifically the double-headed arrow indicating the back-and-forth between the planning subgroup and the whole group study.

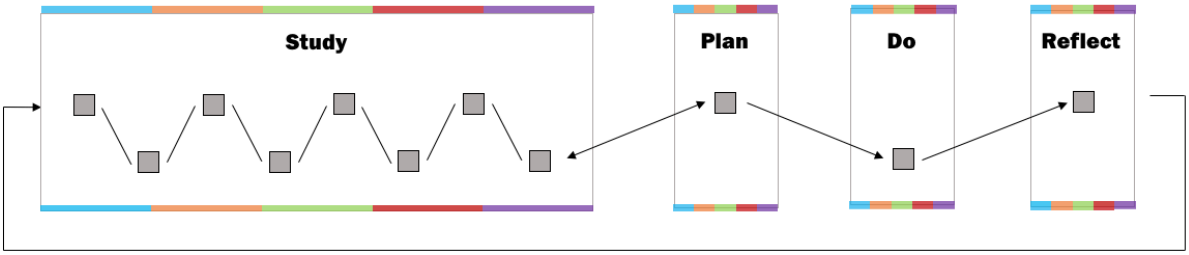

Figure 5. A synthesis of TRU and Lesson Study

In the TRU-LS synthesis, the learning done by the planning team permeates the TLC and vice versa. TRU inquiry cycles thus allow TLCs to leverage the LS process for community-wide development and learning. They also support teachers to develop routines of negotiating their visions of teaching and learning mathematics, essential for effective LS cycles. Finally, they help the learning done in a LS cycle permeate teachers' regular classroom practice.

These two changes to Lesson Study supported by TRU – engaging the whole department in small inquiry cycles within the larger inquiry cycle that is LS and using the TRU framework to scaffold conversations at each stage of the Lesson Study process – help bridge the divide between LS as designed for the Japanese context and the reality of teachers' working lives in the US. They offer the potential to enhance both TRU PD and LS. To TRU PD it adds the benefit of involvement in larger-scale and more intense design than conducted in TRU inquiry cycles, supporting teachers in delving more deeply into content and student thinking. To LS it adds aspects of community building necessary in the US, and the kinds of explicitness and focus on key aspects of instruction that increase the likelihood both of successful lesson design and of collective learning from participating and reflecting on that design.

2. Early Signs of Impact

Over the past three years, the TRU-LS team has collaborated with the Oakland (California) Unified School District to design and implement TRU-LS. Mathematics teachers at OUSD high schools have piloted TRU-LS. Because each school site is unique in terms of size, schedule, personnel and priorities, the implementation of TRU-LS has played out somewhat differently across the various sites. However, the core principles of TRU-LS remain the same and are present in each case.

In this section we summarize instances of practice from three school sites to show how early attempts at TRU-LS are playing out. We selected these examples to highlight consequential differences between TRU-LS and typical LS implementation in the US. Because they are taken from three different school sites, each should be read as independent from the others. For instance, the research theme referred to in Section 2.1.1 is not the same as the research theme referred to in Section 2.2.2. That said, there is notable overlap in themes and research lessons due to similar context. The examples are organized roughly in order of the LS stage in which they take place. Because the study phase of LS is the most extensive and the phase in which there are the most significant changes from LS to TRU-LS, we analyze three examples from that phase. We also analyze two examples from the combined plan, do, and reflect stages.

2.1. Study: Establishing a research theme and theory of action with TRU-LS at sites A and B

TRU-LS integrates inquiry cycles supported by the TRU framework with the study phase of Lesson Study to support teams of teachers to create and refine their research theme and theory of action. Examples from sites A, B, and C indicate how TRU-LS, the inquiry cycles and the TRU framework, helped the teachers select and refine their research themes and theories of action by attending to students' experiences. These examples also show how TRU-LS supported teachers' agency and helped them begin to develop routines of collaborating as a department about problems of practice, building the culture essential to LS.

2.1.1 Site A: Crafting an initial research theme through reflection with the TRU framework

This example shows how a mathematics department gets started with TRU-LS. At site A, the teachers in the mathematics department selected and began refining their research theme by using the TRU framework to reflect on their goals, students' strengths, and areas of possible improvement. Their initial TRU PD inquiry cycles, in this case, collaborative reflections on classroom video, helped focus the discussion on students' experience. The description below illustrates how both the framework and inquiry cycles support the initial study of a research theme.

Building a research theme at site A began with teachers each sharing one goal they set for the school year. Because this site had studied TRU previously, many of the teachers' goals were already framed by TRU. For instance, one teacher stated that her goal was to "find the productive struggle 'sweet spot'" – a key aspect of Cognitive Demand dimension.

As teachers talked, it became apparent that there were three emerging shared themes – supporting student explanations/arguments, building student ownership of their learning, and finding the 'sweet spot' for productive struggle. To further narrow the theme, the teachers discussed strengths and areas of growth for their current students. These were then organized along the TRU dimensions. Teachers voted on the theme they preferred, settling on the 'sweet spot' of productive struggle. Teachers said they saw this idea as connected to both Cognitive Demand issues of Equitable Access, another key dimension of TRU. At the end of the first meeting, teachers agreed to try strategies aimed at supporting productive struggle in their classrooms prior to the next full department PD meeting.

The second department meeting began with the teachers watching and reflecting on a video of students in an Algebra 2 class grappling with a challenging problem. The goal of this activity was

to help the teachers decide what about productive struggle they wanted to focus on. The facilitator prompted the teachers to use the video to think about what productive struggle looks like. The teachers organized their observations about productive struggle in the video using a TRU Cognitive Demand observation tool. They recorded what student behaviors they saw as productive struggle, what factors constrained productive struggle, and what strengths each student brought to the task.

During discussion, teachers noted that they saw students trying an alternate strategy when their current strategy was not working out as they had expected, and asking each other whether a solution or processes "makes sense" – behaviors that they tied to productive struggle. Teachers described two students who put their heads down during the task as "disengaged." This observation prompted a discussion around how to support a student to struggle productively if that student's "immediate response" to a math problem is "I don't get it, I'm done." Many teachers felt that getting students to not shut down was an important precondition for productive struggle. As a result of this conversation, the teachers agreed that mathematical struggle entails positive risk-taking and that students need to feel safe to take such risks. This led to an enhanced research theme proposal: "How do we set up students so that they are comfortable engaging with productive struggle and how do we arrange what they are grappling with so that they can make meaningful progress toward the learning target and standards?"

This narrative illustrates how collaborative reflection on a classroom artifact, buttressed by the shared language of the TRU framework, supported selection and refinement of a research theme during the study phase of TRU-LS. At the initial stages of the TRU-LS study phase, the TRU framework helped the TLC articulate and focus on meaningful problems of practice. Joint reflection on classroom video allowed teachers to deepen their noticing of student experiences. This, in turn, supported refining and negotiating their collective understanding of the research theme.

2.1.2. Site B: Refining a research theme and developing a theory of action through TRU inquiry cycles

The record of practice described below illustrates how ongoing engagement with the TRU inquiry cycles, consisting of experimentation in individual teachers' classrooms followed by collaborative reflection, enriches the study phase of TRU-LS. At site B, the TLC revised its research theme and developed a theory of action by engaging in several TRU PD inquiry cycles of reflection and practice. These cycles helped them develop and sustain the three lenses of *kyouzai kenkyuu* that Fernandez, Cannon & Chokshi (2003) observed that their Japanese colleagues deem critical – the researcher lens, the curricular lens, and the student lens. This narrative describes a department meeting several months into the TRU-LS process in which teachers reflected on artifacts of practice generated when they tried an intervention developed to help them learn about their draft research theme. The TRU framework helped teachers reflect on the artifacts they brought to the meeting and translate their noticing into goals for teaching and learning. These resulted in a revised research theme and theory of action.

The teachers at site B, like those at site A, developed an initial research theme by reflecting on their goals, students' strengths and challenges, and the TRU framework. Their initial research

theme was, “Building student perseverance/capacity to struggle productively, together.” They connected this theme to the Cognitive Demand and Agency, Ownership, and Identity dimensions of the TRU framework.

To engage in the practice part of the TRU inquiry cycle, the teachers chose a teaching strategy that they believed would help them see their research theme of student perseverance “come to life.” After some discussion, they chose a teaching strategy called Three Things: When students are stuck on a math task, the teacher would ask them to state three things they know about the problem and three things they are wondering about. Teachers conjectured that this strategy would help students think about what they already know when they got stuck, and that it might help students become more independent. They hoped that it would prompt students to explain their thinking, and that students might begin using it with each other.

In subsequent meetings, reflections on Three Things and discussions of classroom artifacts related to the theme of perseverance and productive struggle helped the community take up a student perspective on the research theme. Shared observations grounded in the shared language of TRU supported the teachers to build on one another’s thinking about different student experiences of perseverance. The discussion peaked during their fourth department meeting, a little more than a month before their first research lesson.

At the beginning of this fourth meeting, the teachers discussed what happened in their classrooms when they tried Three Things. One teacher began the discussion by connecting her reflection to the TRU “equitable access” dimension. This prompted another teacher to reflect that in his experience, Three Things prompted more students to be more open about sharing their ideas verbally. Another teacher said that she had noticed that while Three Things encouraged students who were “borderline engaged” to talk more, it did not seem to support students who were more physically and audibly disengaged from classroom work. These three observations prompted a community discussion of what is meant by equitable access, and gave rise to the question: Does Three Things help us achieve *equitable* access to collaborative productive struggle? A fourth teacher challenged whether Three Things was actually promoting collaborative productive struggle since the teacher, rather than the students, had provided the strategy.

This challenge led the teachers to more deeply examine each other’s definitions of “persistence,” a word they had used interchangeably with perseverance and productive struggle – but this time with attention to how persistence manifests for students who experience the challenge of mathematics problems differently. In the discussion that followed, the facilitator encouraged them to try to disentangle their different definitions of persistence using the TRU dimensions. They identified more reasons why students might not persevere, tying their observations to each of the Cognitive Demand, Equitable Access, and Agency, Ownership, and Identity dimensions. One teacher suggested that when teachers ask students to share three things when they were stuck, students are prevented from drawing on their “toolbox” themselves. From her perspective, students who failed to persevere because they felt as though they did not have the tools to move forward were not supported to persevere by teachers providing those tools for them; those students should be supported to find the tools they need themselves.

This comment led the teachers to wonder what Three Things gives students access to. While Three Things might give students more equitable access to the *content*, by helping them continue to solve a problem and allowing them to hear strategies developed by other students, it might not give them access to mathematical *practices* including perseverance. Three Things encouraged students to further rely on their teacher to move them forward when they were stuck on a problem. If perseverance meant the capacity for students to struggle productively, *together*, having a teacher ask for three things did not help students develop that capacity *together*.

This articulation of what the teachers really meant by perseverance, tied to their experiences with Three Things and the TRU dimensions, marked a significant step forward in developing a rich research theme tied to student experiences. By the end of this fourth department meeting, the teachers knew that while they cared about supporting perseverance so that students would have equitable access to content, they also cared about supporting perseverance so that students could draw on each other, rather than their teachers, for support during times of need. Moving forward, the teachers oriented towards finding ways to support students to draw on resources such as their peers, notes, or classroom artifacts with less teacher scaffolding.

The teachers' experiences at sites A and B demonstrate how TLC-wide TRU-based inquiry cycles, nested within the study phase of a Lesson Study cycle, supported teachers in developing a deeper research theme that was more directly tied to student experiences, testing conjectures about the impact of teaching strategies together, in advance of their work on independent research lessons, and developing a culture of collaborative inquiry into teaching and learning.

2.2. Plan and Study: Interconnectedness between the planning team and an entire department's study process

One significant difference between TRU-LS and typical LS is the interconnection of whole department PD and the research lesson planning team. In TRU-LS, responsibility for planning the research lesson is assigned to a planning team – but that planning team interacts with the whole department in an ongoing way as it works on critical planning issues. Through this interconnection, *kyouzai kenkyuu* is further developed, particularly with regard to the teachers' development of the curricular and student lenses.

2.2.1. Site A: Refining a lesson plan through a whole department discussion grounded in TRU

At site A, which had selected productive struggle as its research theme, the TLC decided on the topic of systems of equations. The planning team reviewed the progression of related Common Core standards on solving linear equations and systems from grade 7 through high school, examining mathematics tasks they believed would support students to learn the systems of equations content and that would provide students with opportunities to engage in mathematical practices related to productive struggle.

The teacher who volunteered to teach the research lesson told the planning team that he had taught systems of equations the previous year procedurally and wanted to increase cognitive demand this year. Accordingly, the planning team worked through a sample system of

equations assignment from the previous year. This task included the possibility for students to engage in multiple representations, but by and large relied on procedural knowledge.

The planning team then studied three additional tasks that helped illuminate the multiple perspectives students might take when solving systems of equations and the challenges students might experience along the way. However, a tension remained between the team's desire to increase the cognitive demand for students and a fairly narrow interpretation of the content standards. Teachers' interpretation of the standards was such that students only needed to learn how to solve systems using substitution and, only for equations that were in slope-intercept form. This initially constrained considerations around cognitive demand. For example, although one of the tasks was helpful in opening up the space of possible student strategies, it was dismissed as inappropriate because the equations were presented in standard form.

As the planning team completed this series of tasks, they focused on the following question: "What knowledge do we want students to leave this unit with?" Ultimately, they decided that they wanted students to understand 1) that the solution to a pair of equations is the set of points where the graphs of the two equations intersect, and, 2) when and why a system will have zero, one, or infinitely many solutions. The "Classifying Solutions to Systems of Equations" Formative Assessment Lesson (FAL), developed by the Shell Center, was deemed promising in this regard despite its use of equations in standard form. The planning team decided to bring the FAL to the whole department meeting so that everyone could also do the mathematics and aid the team in adjusting the task to hit the productive struggle "sweet spot."

Doing the mathematics together at the whole department meeting generated a fruitful discussion that shaped the planning team's subsequent design decisions. The department agreed that the level of cognitive demand was too high, and pushed the planning team to think more deeply about how to adjust the task without "scaffolding away" the most important mathematical ideas. For example, when the planning team proposed changing all the equations in the FAL to slope-intercept form in an effort to make the task more accessible (echoing discussions that occurred at planning team meetings) one Algebra 1 teacher challenged, "if you alter all these equations to be in slope-intercept form then aren't you giving up a lot of the cognitive demand of this particular task?" Engaging the whole department broadened the collection of possible student strategies for comparing two equations. It expanded the set of connections that might be made, highlighting places where students might engage deeply with the content. Finally, the department also suggested changes that could be made to the task to make it easier for students to access the central mathematics within the allotted time.

In a subsequent planning meeting, the planning team spent significant time anticipating likely student difficulties that could come up during the task. In response to each anticipated difficulty, the team either adjusted the task in ways that did not compromise the key mathematics or planned teacher moves that could support students' continued progress without "scaffolding away" the challenge.

This record of teacher conversations demonstrates ways in which coordination between whole department study meetings and planning team meetings, built into TRU-LS, enriches both the

study and the planning phases of LS. The department's familiarity with TRU provided a common language that enabled the planning team to think more deeply about the mathematics content than their consideration of the standards documents had afforded. Doing the mathematics together in department meetings allowed the teachers to engage more deeply with issues of cognitive demand than if they had discussed it in the abstract, and to be better positioned to understand what transpired in the research lesson. In this way, the synthesis of TRU and LS expanded the opportunities for all department members to learn about the mathematical content and practices through a student lens.

2.3. Do and Reflect: Two different trajectories through the same lesson plan at sites B and C

Sites B and C independently chose to use the same OUSD lesson, the Group Angle Challenge, as the basis for one of their research lessons. The main mathematical activity in the lesson asks students to use their knowledge of angle relationships to find the values of labeled angles in a geometric diagram. Each angle can be determined in two or three different ways. Students are asked to record explanations in a table for how they know the values of the angles. The structure of the lesson moves from students working in groups on the angle challenge and explanation table, to students working in their groups to make a poster reporting on their findings, and finally to students from different groups visiting each other's posters to read and comment. The content focus of the lesson is somewhat limited, but the lesson does offer possibilities for student discourse and argumentation.

The site-based planning teams chose this lesson for different reasons. Site B teachers hoped that the lesson would help students learn to turn increasingly to classroom resources and each other for help instead of to the teacher. Site C teachers hoped that the lesson would help them support students to justify their thinking and critique the mathematical thinking of others. Thus both sites focused on aspects of the Cognitive Demand and Agency, Ownership, and Identity dimensions of TRU. Yet, their different research themes and theories of action led them to make different observations during the research lesson. This, in turn, shaped their post-lesson discussions. In consequence, both sites reflected deeply on their students' experiences, but very differently.

These examples indicate how the non-prescriptive but focused character of the TRU Framework can play out productively in practice. The Group Angle Challenge lesson can be modified in myriad ways, some of which would lead to unproductive conversations. Avoiding this, however, by forcing teachers down any particular path – even if the direction taken is useful – runs the risk of engendering resistance and undermining attempts to build teacher community. That teachers at the two sites were able to pursue their interests was enfranchising, contributing to teacher agency and the ongoing development of teacher community. And, the choices made by the teachers were productive, because TRU problematized practice in focused ways.

2.3.1 Site C: Reflection with the TRU framework supports shifts in theory of action in a subsequent TRU-LS cycle.

Reflecting on their research lesson, site C teachers noted that although students had justified their reasoning to each other while working in small groups on the Group Angle Challenge, much of that mathematical talk did not make it onto their posters or in comments to their

peers. This observation ran against their theory of action. The research lesson teacher had modeled "what critiquing reasoning looks like" and the students were given opportunities to critique each other's work. However, students' critiques did not contain the mathematical thinking the planning team had expected – and that had been observed in the initial small group work.

During their post-lesson discussion, the site C teachers recorded these observations in the context of the Mathematics and the Agency, Ownership, and Identity dimensions. Under Mathematics they wrote, "Students knew how to justify verbally, writing didn't match." They also suggested "Maybe tell them what kind of info to include in the justifications?", thinking that students' mathematical writing might be richer if the teachers were explicit about the criteria they had in mind for richness. Under AOI they wrote "Student critiques were vague," drawing on the AOI dimension's attention to students' critiquing each other's thinking. They added "Students did a good job of holding each other accountable while solving," but "This thinking didn't make it onto the poster."

Their final commentator started by touching briefly on each of the five TRU dimensions, highlighting some of the observations that had been made by the teachers. She then turned to the Formative Assessment dimension, which had not received much attention during the teachers' lesson planning or reflection. After discussing the lesson through the lens of formative assessment she asked about next steps. Her suggestion was to take another look at the team's theory of action in light of some of the teachers' notes documented in the Lesson Study Discussion Guide, Part 2: The Math. Specifically, she drew the teachers' attention to where they had written, "Our goal is for students to be able to 1) think 2) conjecture 3) critique and 4) revise... 'Revise' allows students to know that it's okay to make mindful mistakes." She prompted the teachers to think about how, if at all, this final step of 'Revise' showed up in their research lesson. She drew from the TRU Observation Guide, highlighting two Student Look-fors (Provides specific and accurate feedback to fellow students and Makes use of feedback in revising work) and one Teacher Look-for (Flexibly adjust content and process, providing students opportunities for re-engagement and revision). The final commentator proposed that had the students been given the chance to revise their work, they might have understood why their teacher was asking them to give precise, constructive feedback.

Moving into the second Lesson Study cycle, teachers at site C decided to keep the research theme from their first LS cycle, "building students' mathematical reasoning through student justification." Through reflection on the first research lesson, teachers concluded that tasks presented to students need to be rigorous and open-ended in order to provide students rich opportunities to justify, critique, and revise. The teachers had selected the Group Angle Challenge for their first research lesson because they felt it would provide students opportunities to justify their answers. However, this second pass resulted in teachers becoming more aware of the limitations of the original curricular task: because there was only one correct answer for each angle measure and only one or two ways to explain that answer, students' justifications were not as central to the task as they could have been. One teacher suggested that "more rigorous open-ended tasks" would offer more opportunities for students to share a variety of justifications and critiques.

In response to the research lesson final commentary and their collective reflections, teachers adjusted their theory of action for their second LS cycle to incorporate the four-step justification process highlighted in the final commentary. They used a TRU Formative Assessment lens when reflecting on instructional strategies, students' learning experiences, and task design in preparation for their second research lesson.

This sequence of events illustrates how the TRU framework supported the reflection phase of TRU-LS. At site C, the external commentators explicit focus on Formative Assessment supported the community to reason about their research lesson observations more deeply, and ultimately focused their attention on richer mathematics.

2.3.2 Site B: What would support students to turn to each other as resources? A focus on Agency, Ownership, and Identity

Reflecting on their research lesson, site B teachers and their final commentator focused largely on the ways in which students asked for help from each other and their teachers and the role that teacher interventions played in student perseverance. Site B teachers discussed how students had unevenly drawn on resources that the teachers had planted around the room to help them solve the angle challenges. Some students readily took out their notes and referred to posters they had previously made that described different types of important angle relationships. Others, however, did not, and seemed at a loss for how to proceed without heavy scaffolding from a teacher. These observations also challenged the site B teachers' theory of action. The resources and questions that they had developed had supported some students to struggle productively together, but not all of them.

Site B teachers did not explicitly categorize their observations into the TRU dimensions, but the Cognitive Demand, Equitable Access, and AOI dimensions appeared in their language. The final commentator encouraged them to focus on the AOI dimension, especially with respect to how teacher interventions could both enhance and undermine student agency during group work. She highlighted observations in which teachers had described having to scaffold student thinking more than they felt was appropriate for their goals.

She also discussed an observation she had made that she felt had implications for a student's agency. She had heard the student talk about an idea with confidence and certainty when the teacher wasn't present. But when the teacher dropped by to check in, the student had asked a question about that idea as if she hadn't already thought of it. The teacher, unknowingly, went over some of the same ideas the student had already reasoned through. When the teacher left, the student attempted to reproduce the teacher's phrasing of an idea that had previously been her own, and she seemed to lose track of her prior thinking. This kind of interaction has the potential to undercut the student's agency.

These observations deepened the conversation about AOI. Site B teachers reflected on the implications of their interventions during group work, given that high school students' mathematical agency might be fragile despite a teacher's best efforts. The final commentator also encouraged the teachers to think of ways that they might broadcast the resourceful mathematical work being done by some groups to others, so that students could learn about using resources from each other rather than their teachers.

In subsequent TRU-LS cycles, site B teachers focused further on helping their students develop tangible mathematical resources that they could draw on during group work. They also attended more carefully to the ways in which students were supported to engage with the big mathematical ideas in their lessons. They conjectured that students would be better positioned for collaborative productive struggle if the mathematical ideas present in a lesson were worth the struggle and if available resources engaged students with those ideas.

At sites B and C, the teachers began by focusing on different aspects of student engagement with the mathematics. This initial focus on mathematics practices – drawing on resources at site B and explaining and critiquing thinking at site C – led the teachers to reflect more deeply about the affordances of the mathematics itself and of their own interactions with the students. Ultimately, their discussions over multiple cycles span all five dimensions of TRU.

3. Discussion

For any form of professional development to take hold and flourish, it must mesh with the context in which it is to be implemented. In Japan, Lesson Study is deeply embedded in supportive school contexts and structures. Teachers are explicitly provided time and space for collaboration, and are supported in working together; there is a decades-long tradition of attending to content and practices in instruction and professional development; and, curricula provide a rich and stable base for curricular inquiry. Under such circumstances, Lesson Study can function as intended, supporting the continuous development of Japanese teachers' knowledge and practices. The US context for professional development differs substantially. Although conditions do vary, as a rule teachers in the US rarely have opportunities for collaboration. Teaching practice is largely considered private, and when external observers walk into a classroom, the purpose is often evaluative. Curricula vary substantially in the affordances they provide both for content and practices, and there is much less of a tradition of attending to student thinking than in Japan. TRU professional development was designed in and for the US context, with explicit attention given to community building among teachers and with a structure that helps teacher to focus on key aspects of instruction while not being prescriptive in ways that would undermine teacher agency.

A primary affordance of LS is the learning that comes from teachers' extended study of important mathematical ideas and the ways that students make sense of them. A primary affordance of TRU is the explicitness with which it highlights productive aspects of the learning environment, with a focus on the students' experience of the mathematics. The five dimensions of the TRU framework provide an explicit language for focusing on essential aspects of classroom practice. This has been useful across the US, providing grounding for professional development efforts that might otherwise be scattershot. In addition, the TRU framework helps to make explicit the goals and theories of action for research lessons and observations of them – something essential in the US but also helpful in Japan, where key aspects of practice are largely tacit and passed on by tradition. The use of TRU inquiry cycles, which bring aspects of research lesson design to the entire teacher learning community, helps to prepare all members of the TLC to profit from observing the research lesson and hearing the lesson commentaries. It also supports the development of teacher community. In these ways, TRU-LS provides a mechanism for the functional enrichment and adaptation of LS to the US context.

The TRU-LS work is in its early stages, and the evidence presented in Section 2 is preliminary. This is as it must be: it takes time to build supportive contexts, and the development of teacher knowledge and expertise is, like all expertise, a process that takes thousands of hours of concerted practice and reflection. However, the evidence thus far suggests that there is some reason for optimism.

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