LEARNING WITH AND FROM TRU: TEACHER EDUCATORS AND THE TEACHING FOR ROBUST UNDERSTANDING FRAMEWORK

INTRODUCTION

The Teaching for Robust Understanding (TRU) framework delineates five essential dimensions of classroom practice. The framework and tools developed to support its use are used in a range of teacher learning communities, both pre-service and in-service. TRU is not prescriptive, leaving great latitude for teacher educators in its implementation – and thus for teacher and teacher educators’ learning. This chapter begins with an outline of the framework and its affordances. The bulk of the chapter contains descriptions by teacher educators who have developed and worked with TRU of the ways in which they have, by virtue of their involvement with the framework, developed deeper understandings of ways to support teachers, and their own conceptions of teaching and teacher education.

The key idea undergirding TRU is that in order for students to emerge from mathematics classrooms as knowledgeable and flexible thinkers and problem solvers, the classrooms must offer significant opportunities along the dimensions described in Figure 1. Those five dimensions are necessary and sufficient. If a classroom does well along all of the dimensions in Figure 1, students will emerge as powerful disciplinary thinkers; if there are significant difficulties in any of the five dimensions, many students will not (Baldinger, Louie, and the Algebra Teaching Study and Mathematics Assessment Project 2016; Schoenfeld 2013, 2014, 2015, 2018; Schoenfeld and the Teaching for Robust Understanding Project 2016).

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1 In K. Beswick (Ed.), International Handbook of Mathematics Teacher Education, Volume 4, The Mathematics Teacher Educator as a Developing Professional (pp. 271-304). Rotterdam, the Netherlands: Sense publishers.
In essence, TRU provides *principles* for powerful learning environments. TRU does not prescribe particular ways of teaching; rather, it provides a growing set of tools for problematizing and reflecting on teaching, with an eye toward enhancing teaching practices and classroom environments along the five dimensions described in Figure 1.

As such, TRU provides pre-service and in-service teacher educators a great deal of latitude – the central question being, “how can I use the ideas in TRU to help teachers, coaches, and teacher learning communities develop richer understandings of teaching that enable them to produce more powerful learning environments for their students?” Addressing this question provides multiple learning opportunities. In using TRU, teacher educators are called upon to reflect on the nature of powerful learning environments, and to develop mechanisms for helping teachers learn more deeply. Because there is latitude in implementing TRU (there is no prescribed “right way” to teach, and there are many ways to enhance current teaching practices along the TRU dimensions), teacher educators working with TRU have the latitude to adapt TRU to their local contexts and to build tools for them. Indeed, “early adopters” – better called “early adapters and developers” – have helped to shape the collection of available TRU tools (see

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**The Five Dimensions of Powerful Classrooms**

<table>
<thead>
<tr>
<th>The Content</th>
<th>Cognitive Demand</th>
<th>Equitable Access to Content</th>
<th>Agency, Ownership, and Identity</th>
<th>Formative Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td>The extent to which classroom activity structures provide opportunities for students to become knowledgeable, flexible, and resourceful disciplinary thinkers. Discussions are focused and coherent, providing opportunities to learn disciplinary ideas, techniques, and perspectives, make connections, and develop productive disciplinary habits of mind.</td>
<td>The extent to which students have opportunities to grapple with and make sense of important disciplinary ideas and their use. Students learn best when they are challenged in ways that provide room and support for growth, with task difficulty ranging from moderate to demanding. The level of challenge should be conducive to what has been called “productive struggle.”</td>
<td>The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core disciplinary content being addressed by the class. Classrooms in which a small number of students get most of the “air time” are not equitable, no matter how rich the content: all students need to be involved in meaningful ways.</td>
<td>The extent to which students are provided opportunities to “walk the walk and talk the talk” – to contribute to conversations about disciplinary ideas, to build on others’ ideas and have others build on theirs – in ways that contribute to their development of agency (the willingness to engage), their ownership over the content, and the development of positive identities as thinkers and learners.</td>
<td>The extent to which classroom activities elicit student thinking and subsequent interactions respond to those ideas, building on productive beginnings and addressing emerging misunderstandings. Powerful instruction “meets students where they are” and gives them opportunities to deepen their understandings.</td>
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**Figure 1. The five dimensions of powerful classrooms**
LEARNING WITH AND FROM TRU

http://TRUframework.org) and our collective understandings of ways to support teacher development using TRU. At the same time, TRU’s openness places demands on teacher educators. The challenge of figuring out how to make effective use of the principles in any particular professional environment can be substantial. In this chapter, six sets of early adapters/developers briefly describe their work with TRU and reflect on their learning as a result of that work. Each group was asked to address the following:

a. Who you are, and what your responsibilities are,
b. How you came to encounter TRU,
c. What your learning trajectory has been, including
   i. what you find easy or challenging, in theoretical terms. (How has your understanding of TRU developed? This may well include how you use TRU as a tool with teacher learning communities.)
   ii. what you find easy or challenging, in practical terms. What issues do you face as you try to help members of a teacher learning community get their heads around TRU?
d. Other issues you want to raise/discuss.

Following those descriptions, we discuss the challenges and opportunities of working within this kind of framework. We note that many of our discussions end with questions or unresolved issues. That is because we as teacher educators are grappling with those issues as we move forward. Our learning is an ongoing process.

EARLY DEVELOPMENT AND SUBSEQUENT TENSIONS
(NICOLE LOUIE AND EVRA BALDINGER)

We were part of the research team that initially developed the TRU framework. As the team’s attention turned to teacher professional development, we were charged with developing tools that would be more conducive to supporting teacher learning and less likely to be misused for high-stakes teacher evaluation than the rubrics we had been working with for research purposes. Drawing on our experiences as mathematics teachers and instructional coaches as well as current research, we developed the TRU Math Conversation Guide1 (Baldinger, Louie, & the Algebra Teaching Study and Mathematics Assessment Project, 2014).

Our primary goal in designing the Guide was to support teachers and the professionals who work with them (e.g., coaches, administrators, and colleagues) to leverage TRU to nurture collaborative relationships, building on teachers’ own concerns, goals, and strengths to advance collective learning. We drew from the premise that teachers’ most meaningful learning toward ambitious teaching occurs when they work with others, coordinating diverse perspectives and expertise to investigate problems of practice (Cabana, Shreve, & Woodbury, 2014; Horn &
As a result, we framed the Guide as a set of key questions for a teacher learning community (or a teacher and coach) to think about during planning, execution, and review of a lesson. For example, the core questions for Dimension 4 of the TRU Framework, *Agency, Ownership, and Identity*, are: “What opportunities do students have to see themselves and each other as powerful doers of mathematics? How can we create more of these opportunities?” These are elaborated as points of discussion for teachers with a set of questions that begins as follows:

- Who generates the ideas that get discussed?
- What kinds of ideas do students have opportunities to generate and share (strategies, connections, partial understandings, prior knowledge, representations)?
- Who evaluates and/or responds to others’ ideas?
- How deeply do students get to explain their ideas?

Here, we discuss two considerations that came into our design of the Conversation Guide: (1) supporting focused and coherent learning over time, and (2) supporting teacher agency and choice. Although many other ideas influenced our design, we focus on these two because of their relationship to questions that have emerged for us since the Guide’s initial release. Our aim is less to explain or justify our choices than to bring readers into some of the spaces in which we ourselves are still wondering and learning.

**Supporting Focus and Coherence Over Time**

Research has consistently identified focus, coherence, and duration as essential for effective professional development (Darling-Hammond, Hyler, & Gardner, 2017; Desimone, 2009). These qualities can be difficult to achieve, however. Setting aside structures that promote one-off workshops and trainings, the complexity of teaching itself can lead teachers and their partners to jump from one idea to another, without making clear progress in any particular direction. In the midst of “the blooming, buzzing confusion” of classroom life (Sherin & Star, 2011, p. 69), we have seen that a TRU focus can counter that tendency. A coach in the Oakland Unified School District described an instance of this in her work with a teacher:

She [the teacher] had a lot of thoughts swirling in her head. … [The Conversation Guide] focused her and she was able to pick some questions and some ideas that she wanted to talk about. … When she started to fly off and get in her head, we were able to reground her and call her back to these questions.

In addition to focusing individual conversations on issues at the heart of teaching, the dimensions have the potential to serve as a backbone that organizes teacher learning and lends it coherence over an extended period of time, across multiple
contexts. For example, we have seen a district adopt a focus on Agency, Authority, and Identity (Dimension 4), and invest substantial resources over the course of several years to support teacher collaboration around creating opportunities for students to see themselves as powerful doers of mathematics (see Section 5 below). Spending this amount time with a consistent focus has been essential for building a shared vision and capacity to enact that vision.

Naturally, we experienced tensions associated with focus and coherence. One is the potential for tunnel vision, losing sight of important aspects of teaching and their connections in favor of one particular piece of the puzzle. TRU’s five dimensions span a broad range of concerns, and in the Conversation Guide, we attempted to craft questions that draw out connections and overlap between areas. Whether or when this works well is an open question. There are also tensions between fostering focus and coherence and supporting teacher agency and choice, as we discuss below.

Supporting Teacher Agency and Choice

Professional development (not to mention public discourse) often positions teachers as deficient and in need of fixing. We are committed to a contrasting perspective that highlights the knowledge, goals, and strengths that teachers bring to their work. The TRU Conversation Guide reflects this in a number of ways, including open-ended questions that invite educators with diverse experiences, background knowledge, and roles to all contribute to collaborative learning. We also aimed to give teachers room to direct the focus of their learning, which TRU’s range facilitates. Teachers can choose a dimension (or part of a dimension, or an intersection between multiple dimensions) that speaks to them. Some teachers might choose to focus on developing “more meaningful connections” between facts, procedures, and important ideas and practices (see Dimension 1: The Mathematics); others might work on creating more opportunities for students “to see themselves and each other as powerful doers of mathematics” (see Dimension 4: Agency, Ownership, and Identity). Honoring those choices is an important way of respecting teachers’ goals, commitment, and intelligence, and of developing the trust necessary to support relationships that in turn support ongoing learning. As one coach who has used the Conversation Guide has said,

[Teachers] feel like they’re constantly being told, “You’re doing it wrong. Do this. Why isn’t this strategy being implemented?” [Conversations with the Guide can be] more about … things that they want for their kids and things that are important to them.

Supporting teachers to focus on “things that they want for their kids” may be in tension with supporting system-wide coherence, however. In some cases, we have seen teachers with varied concerns find space to exercise agency within a TRU focus that has been externally defined (e.g., a focal dimension chosen by their
district). But in other cases, teachers have experienced mandates to connect to TRU as yet another directive, inconsistent with their sense of professional autonomy. One challenge we continue to face, both in the Guide and in PD sessions, is how to orient conversations in productive directions without being directive. In the guide we can point to important issues with our questions (see the examples above). But, because TRU is not prescriptive, the challenge has been to take teacher concerns and frame them in ways that can be addressed productively. For example, the problem “my students don’t persevere” can be re-framed as an issue of formative assessment and cognitive demand: “What do you know about their thinking (formative assessment)? Can you offer them challenges within their capacity for productive struggle (cognitive demand)? With some experience succeeding at this, support in reflecting on it, they might come to persevere more.” Developing and refining the capacity to do such framing is an ongoing issue. We are getting better, individually, at recognizing and managing such tensions – but we do not yet have ways to build such support into our materials. Additionally, TRU does not include everything teachers might legitimately wish to focus on. Teachers might, for example, want to focus on developing strong relationships with students, partnering with families, and supporting students to understand and critique social injustices using mathematics, none of which are easy to locate in TRU. It has become clear that fleshing out TRU tools to make such connections will be an ongoing challenge. These tensions surrounding agency also raise questions with implications beyond the learning interactions that we had in mind when we wrote the Conversation Guide. How might a focus on teacher agency and choice on a larger scale – for example, in selecting, adapting, or authoring frameworks for teaching and learning, or in structuring the work day – create opportunities for teachers to build on their commitments and experiences to inspire and nurture their personal growth as professionals, and the growth of their profession?

When and How to Name Oppression

A third area that has raised tensions and questions for us regards when and how to name oppression. When we first wrote the Conversation Guide, supporting teachers to investigate and transform inequitable classroom interactions was very much on our minds. Yet there is only one sentence in the Guide that makes oppressive power dynamics explicit. That sentence is part of a suggestion to ground conversations in specific, detailed evidence of student thinking and strengths. It reads:

Attending to particular students can help us think about patterns of marginalization in society at large (e.g., fewer resources for ELLs, or stereotypes that link race, gender, and mathematics ability), and how our
classrooms might work to replicate or counter those patterns for our own students. (p. 5).

This sentence is buried in the Guide’s front matter, which many users look at infrequently if ever. In other parts of the Guide, we posed questions asking, “which students” participate and how, and how more opportunities could be created for “each student.” Our observations suggest that this language fails to support teachers and others to engage in conversations about race, class, gender, and other lines along which privilege and oppression are organized (as critics of “math for all,” e.g., Martin, 2003 might have predicted). Instead of encouraging these conversations, our uses of “which students” and “each student” seem to perpetuate the taboo of naming social hierarchies. We have observed instances of teachers talking around these hierarchies, seemingly uncomfortable with confronting them, as well as instances of teachers casually reproducing damaging assumptions about who is capable of doing what. How could explicit and frequent naming of our concern for Black, Latinx, and indigenous students, in the main text of the Conversation Guide, shape teachers’ opportunities to examine oppressive power dynamics in their classrooms? What would be the tradeoffs in using words like “minoritized” or “nondominant,” which are more adaptable to local conditions of privilege and oppression but also more easily taken up in ways that do not challenge those conditions? How could explicit naming of oppression be done in ways that are consistent with foregrounding teachers’ questions, strengths, and goals and nurturing collaborative, collegial relationships between teachers, their colleagues, and those who are often in positions to evaluate them (such as administrators and coaches)?

Concluding Thoughts

We have always known that the real value of the TRU Conversation Guide would rest not in the document itself but in the hands of the educators who would pick it up, adapt it, and use it. It has been exciting and thought-provoking to see what people in Oakland, San Francisco, Chicago, and elsewhere have done with it. These experiences have prompted new learning for us as teacher educators and scholars of teacher learning. We look forward to continuing to learn about the tool, how it interacts with different educational systems, and how it might be productively adapted to support more powerful work. Three main take-aways from our work as teacher educators have been that: (1) collaboration with those “in the trenches” – both district coaches and teachers – is essential to build and refine tools that have ecological validity; (2) theory can help drive practice in productive ways, if the two live in synergy; and (3) extended work of this type is very much context-driven and context-sensitive, requiring sensitivity to the needs of teacher learning communities while remaining focused on key aspects of professional development.
We are still learning, individually and collectively, how to deal with the tensions identified here.

TRU IN THE MASTER’S AND CREDENTIAL IN SCIENCE AND MATHEMATICS EDUCATION PROGRAM

(JACOB DISSTON)

I direct the Master’s and Credential in Science and Mathematics Education (MACSME) secondary teacher education program at the University of California, Berkeley, a program that supports prospective teachers in earning both a post-baccalaureate teaching credential and a Master of Arts degree in Education. Our aim with TRU is to develop a shared structure and language for talking about teaching and learning, both in planning for instruction and reflecting on and analyzing instruction. Here I describe ways we have worked to integrate TRU and TRU-related tools into the parts of the program that primarily support students’ fieldwork: our Supervised Teaching Seminar, our Supervisor Student Teaching Observation Form, our Teaching Methods Course.

The most meaningful and transformative PD experiences I experienced in my 17 years as a middle school mathematics teacher were those that involved collaboration with colleagues, and those that supported my development over an extended time period – specifically, working with student teachers and Lesson Study. Both positioned me as a collaborator and co-learner, working to investigate teaching and learning in real classrooms. Partnering with prospective teachers called for being explicit about teaching decisions, in planning and in the moment; student teachers and I could examine the intentions and understand the implications of choices we make on student engagement and learning. We explored how the collection of individual decisions come together to support structures and classroom norms, and examined whether and how these structures support the types of student engagement and learning that we wanted. Lesson Study creates opportunities to collaborate in depth on lesson planning, and to gather data to understand the impact of the lesson.

My experience in working as a supervising teacher and in Lesson Study prior to having TRU is that a lot of time and effort is spent searching for words and structures to describe what is taking place. With the TRU framework and vocabulary, the work becomes sorting what’s going on into the five dimensions and building a consensus about the effects of moments and events within the lesson on student learning with regards to the five dimensions.

What follows describes 3 structures we’ve developed to introduce and integrate the TRU framework into our work with prospective teachers.
**TRU Video Jigsaw**

Our first challenge has been to orient prospective teachers to TRU quickly. Over time, we evolved the following “immersion” strategy. The first day of the MACSME program prospective teachers watch a 5-minute video clip of a classroom lesson. We begin by having prospective teachers examine the task that is the focus of the video, asking them to work it and then to anticipate all the ways that students might approach it. They reflect on approaches that lead to complete and correct answers and those that don’t – all of which illuminate student thinking and understanding. We then introduce TRU and the Observation Guide, splitting the students into 5 ‘home groups,’ each focusing on one TRU dimension. Prospective teachers reflect individually on how ideas in their focal dimension connect to their experiences as school students and their early experiences tutoring or teaching. They discuss their experiences within groups to see how their understandings do or don’t align, and what they anticipate seeing in the video of the teaching episode.

When they watch the video, the prospective teachers write down everything they see that’s connected to their chosen dimension. They are asked to avoid judgmental observations and to focus on how to sort observations into one or more of the TRU dimensions, identifying the evidence from the video supporting their decisions (e.g., “The point at which the teacher turned her attention to the female student who was leaning back and not participating in the group’s conversation seemed important for Agency/Ownership/Identity because…”).

Members of each dimension home group then take turns sharing what they noticed, supporting observations with evidence from the video. They add other group members’ ideas to their own lists so that they can represent their group’s complete set of observations once we switch to jigsaw groups.

Jigsaw groups composed of at least one member from each of the home groups share observations and evidence one at a time, in the order given in Figure 1. The group then opens up the discussion, comparing what was noticed through each of the lenses. It becomes clear that different events and moments in a lesson stand out and have different significance depending on which TRU lens you are looking through, and that sharing these different perspectives helps everyone in the group deepen their understanding of classroom dynamics.

The class then reflects collectively on the experience, focusing on what TRU seemed to surface that might not otherwise have been noticed and what, if anything, happened in the clip that was left out of the discussion and was missed – whether there are aspects of teaching and learning that may not be accounted for in the TRU framework. This surfaces tensions we are still dealing with. Prospective teachers commonly point out that issues related to classroom management and time management don’t get discussed, even though they seem to feature prominently in the classrooms in which our prospective teachers observe and teach. Negotiating the tension between key points of focus and their everyday concerns is a challenge.
(Of course, the sooner they think big picture, the sooner management issues begin to get resolved.) We have also noticed that issues related to how race and gender play out in classrooms don’t feature in the video jigsaw activity discussions. A question for us to consider is whether these issues are absent in the jigsaw discussions because the TRU dimensions somehow focus our attention elsewhere, or because the video clips we’ve picked either don’t feature these issues or video doesn’t portray these issues accurately enough. The video and live classroom observations create vantage points and opportunities to explore aspects of teaching that ultimately deepen our ability to analyze events in the classroom so that we can better support student learning. There are still questions: Does TRU guide or force us to see certain things? Or do we only see certain things, and then try to sort them into the TRU dimensions?

*The MACSME-TRU Supervisor Observation Form*

Another challenge was to support observations of practice teaching in a way that keeps TRU central. We developed a TRU-based observation form to organize observation notes and feedback to student teachers. The form includes identification information about the placement, a description of the teaching practice goal the prospective teacher is currently working on, a description of the lesson activities and links to the lesson plan. These are filled out by the prospective teacher and shared with the supervising teacher prior to the observation visit. The observation section of the form, used by the supervisor to record observations during the lesson, is divided into 6 cells: one for each dimension and a cell for general comments.

During the lesson, the supervising teacher makes notes and sorts them into the cells according to which TRU dimension or dimensions are most appropriate. Depending on the student teacher’s goals for the day and what feedback she has asked for, the supervisor may focus on one, a select few, or all TRU dimensions. The supervising teacher makes decisions in real time about what to record, and where, in the observation sheet. This act of sorting significant moments and aligning them to specific TRU dimensions puts the supervisor in a more active mode of observation.

Afterward, the supervising teacher and prospective teacher meet to reflect on the lesson. Typically, the prospective teacher begins by reflecting on how she feels about the lesson generally, and then what aspects of the lesson stand out as interesting or important, and how these aspects align to and are informed by the TRU dimensions. The supervising teacher can respond by building on the prospective teacher’s reflections, adding details and evidence they collected which supports the alignment the student teacher has made. Or, the supervising teacher can ask questions or make statements that surface important aspects of the lesson which the prospective teacher has not mentioned.
The TRU framework grounds the reflection in common vocabulary and provides a structure for reflecting on a lesson, both in terms of what worked well and what could be improved. But because the supervising teacher is the one sorting the significant moments into the TRU dimensions, we added a step in the process to include a response to feedback from the prospective teacher following the debrief discussion: prospective teachers write responses in the shared observation document, sharing their thoughts about the 5 dimensions with regard to the lesson itself, the debrief discussion, and the supervising teacher’s written notes. We have also experimented with a process for debriefing an observation in which the supervising teacher presents a set of interesting and significant moments with the prospective teacher, one at a time, and asks the prospective teacher which dimension(s) of TRU she feels it aligns to. In this way, the prospective teacher participates in the process of examining the notes from the lesson enactment and exploring the possible alignment to the TRU dimensions, rather than accepting the alignment that the supervising teacher felt was strongest.

**TRU as a focus for weekly reflective journals**

All MACSME students are enrolled in the Supervised Teaching Seminar in which we discuss issues that arise in their field placements. Each week, prospective teachers respond to a journal prompt on our online class portal by Friday night. The group reads and comments on each other’s posts over the weekend, and the collection of posts serves as the foundation for our weekly discussion. The weekly journal topics help to guide what our prospective teachers focus on during fieldwork, in observing lessons their Supervising Teachers teach, and during lessons the prospective teachers lead themselves. By focusing on each of the TRU dimensions for a week or two, prospective teachers become experienced in looking through a specific lens to identify the moments in a lesson that align to that dimension. We also ask them to focus on connections between the dimensions, and aspects of teaching and learning that fall within the intersection between dimensions, as a way to explore how certain lesson structures or teacher moves might be leveraged to achieve particular teaching practice goals, or might help identify potential pitfalls and compromises that can inhibit student engagement and learning. For example, one of our journal prompts in the fall semester asked prospective teachers to observe their classrooms through the lens of Agency/Ownership/Identity, and to make connections to the Access dimension:

Last week we focused on looking for moments in the classroom that connect to Agency/Ownership/Identity …

This week continue to observe/reflect on issues related to Agency/Ownership/Identity - especially in how those issues relate to
Equitable Access: if we are working to structure things so that all students have access, why aren’t they all engaging?

In this way prospective teachers can question what specific elements of lesson structure and teacher moves, besides those related to creating equitable access for students, might be necessary for teachers to consider in order to support all students in developing productive identities as learners. Similarly, we can examine through the intersection of Access and Cognitive Demand how structures that support improved access might serve either to diminish or maintain the cognitive demand of an activity and the potential for students to engage in productive struggle. These intersections between dimensions have proven to be a productive way to develop a deeper shared understanding of the TRU dimensions, and to identify aspects of a classroom episode that otherwise might be missed.

What we’ve learned about TRU, and the questions that have arisen

TRU provides a useful structure for investigating teaching and learning within a professional learning community like MACSME. The TRU framework as experienced through the structures described in this chapter provides a means to establish a common vocabulary and a set of lenses for identifying what’s important to notice and examine in an episode of teaching and learning, and for planning and reflecting on instruction in ways that supports the development of effective teaching practices.

The structures described above resulted from some years of experimenting with how best to introduce TRU to prospective teachers whose main experience of teaching has been through the “apprenticeship of observation.” They do help prospective teachers re-orient to classroom phenomena, but they are still a work in progress. Questions we will examine in future work include: What, if anything, do we miss when we look at teaching and learning through TRU? What is the difference between looking at a classroom through a specific TRU lens, where we only pay attention to moments that seem aligned to the one dimension, and looking through all of them at once, picking out moments that seem important, and sorting them into the TRU dimensions? And in what ways do the individual TRU lenses, and the intersections or combinations of multiple TRU lenses, help us notice and understand more nuanced aspects of teaching and learning?

TRU MATH IMPLEMENTATION BY THE SILICON VALLEY MATHEMATICS INITIATIVE

(DAVID FOSTER AND TRACY SOLA)

Supporting productive shifts in mathematics instruction is challenging. Dominant belief systems, counterproductive federal, state, district and school policies, a long tradition of “demonstrate and practice” pedagogy, and historically low expectations
for under-represented students are just some of the obstacles that must be overcome.

The Silicon Valley Mathematics Initiative (SVMI) (See http://www.svmimac.org/home.html) has been working to support improved mathematics teaching and learning since 1996. Participating districts receive year-round professional learning, a formative and summative performance assessment system, funding to support district mathematics coaching, and a network including meetings and workshops with mathematics teachers, leaders and administrators.

Our purpose has been to describe a new vision of teaching and learning and to share innovative instructional methodologies to improve instruction. Until recently, communicating all the necessary elements for a program of sustainable change has been an immense challenge. We had long lists describing mission statements, goals, a range of loosely connected programs, and a set of strategies and improvement plans. To create a common vision, SVMI facilitators provided readings, cited important research, developed lengthy bullet points of actions, and created bibliographies of books and papers for reference.

As an illustration of the challenge consider NCTM’s newest guide to practice, Principles to Actions. The book has tons of lists – lists for mathematics teaching practices (8), beliefs about teaching and learning mathematics (6), establishing mathematics goals to focus learning (7), implementing tasks (9), using mathematical representations (12), levels of classroom discourse (24 cells), facilitating mathematical discourse (8), posing purposeful questions (8), building procedural fluency from conceptual understanding (9), supporting productive struggle (8), etc. For anyone without extensive experience, these lists are overwhelming and hard to embrace. Creating coherent professional learning activities out of these lists is at best challenging and at worst unwise, since teaching appears fragmented when described in lists. What we needed was a comprehensive and concise vision of mathematics teaching and learning.

TRU came about at an opportune time. American education was focused on shifting to the Common Core State Standards for Mathematics (CCSSM) and districts and schools were intent on learning about CCSSM. Standards allowed teachers and leaders to focus on mathematics content but offered little help regarding ways to describe or create mathematically powerful classrooms. That is what TRU does, in efficient and coherent form. Being able to organize our work around the five TRU dimensions, with confidence that all key issues can be addressed, allows for much more efficient professional development. SVMI has rolled out TRU Math in in Northern California, Southern California, the Greater Chicago Metro Area, and New York City. It is core to our work.

Introducing TRU

A TRU launch begins with formal presentations at our Math Network Meetings, where mathematics leaders, mathematics coaches, and teachers on special
assignment meet regularly for professional development. We show a video of a highly engaged classroom and ask participants to list the attributes they observed in the video lesson. One or more of five scribes at easels in the front of the room record the comments the participants make. Once the chart papers are full, the scribes reveal the categories they were using to record comments – the five dimensions of TRU. Since every attribute landed on at least one of the chart papers, the five dimensions encompass the entire instructional domain. The fact that some attributes landed on multiple papers, indicated the differences between dimensions but some overlap. The overlaps show connection between dimensions such as Access and Agency or Cognitive Demand and Mathematics.

TRU underpins all SVMI professional development. When working with district superintendents and curricular administrators at our tri-annual meetings, addressing site administrators during our Principal as Instructional Leader Meetings, working with mathematics leaders at our Math Network Meetings, and presenting professional development to teachers during summer institutes or school year follow-up professional development workshops, the five dimensions of TRU frame professional learning and provide the lens to reflect upon student learning outcomes. For administrators, TRU highlights the kinds of things that need their support. For teachers, TRU focuses on the aspects of instruction over which they have control.

**Going into depth**

As we continued working with educators to deeply understand TRU, we realized the usefulness of focusing sessions on a single dimension. An early favorite was Agency, Authority/Ownership, Identity. We assisted educators in exploring those terms by focusing on the root words of the terms. Unpacking what it means to be an agent and identifying attributes of an agent, helps teachers make sense of agency (being self-reliant, a self-starter, responsible for others, being in the role of a leader...). The root word of authority is author, as in author of ideas. Shifting the term authority from merely the concept of a central power to the creator of thought invites educators to see students in a different but very important role – facilitating students as creators of thought is challenging and nuanced, inviting and requiring a different role for the teacher. The term identity was often more accessible to the participants, especially with recent emphasis on growth versus fixed mindset. Using these characteristics, the role of students in mathematically powerful classrooms began to take shape. We then engaged the educators with videos of classrooms where students were actively discussing important mathematical ideas, challenging each other’s thinking, sharing ideas and strategies, and clarifying their understandings. The use of classroom video was an important tool for promoting collegial discussions and assisting teachers and leaders in deeply defining Agency, Authority, and Identity. The next levels of discussions, prompted by videos, were about how to create a classroom culture to foster these important characteristics in
students. This opened the door to sharing routines and activities that promote and sustain classroom discourse. Mathematics or number talks, cooperative logic activities, group discussion protocols, group work quizzes, think-pair-share routines, sentence frames and sentence starters, are just some of the instructional techniques we introduced and promoted to build Agency, Authority and Identity in our students. These techniques shift the heavy lifting of learning from a teacher who is expounding to a facilitator who fosters learning. The students become the owners of their own learning and resources for one another.

The Equitable Access dimension promotes a social justice agenda, taking on achievement and opportunity gaps directly. At the heart of this dimension are belief systems and student expectations. Our professional learning in this dimension began with conversations and activities that surfaced and confronted belief systems. Using readings and discussions raised awareness. Observing classroom videos that illustrate students, whose capabilities are underestimated, struggling, persevering, and succeeding helped to confront traditional beliefs. Engaging teachers in closely examining student work and conducting consensus scoring sessions created a space to share and negotiate common values about student expectations. Setting goals and creating actionable plans was often a next step for teachers and leaders.

We read about and discussed the pernicious impact of tracking. Another factor that often denies students’ access centers on language. Traditionally, English Learners are often prevented or “protected” from engaging in rich tasks, or tasks that require negotiating a written or real-life context. Instead of moving away from language-rich mathematical problems and tasks, teachers need to create opportunities for English Learners to engage productively with mathematically and linguistically rich tasks. We introduced routines such as three reads, problem stems, close reads, and video contexts to create accessible strategies enabling English Learners to tackle rich mathematical tasks. Our professional development included an emphasis on students’ explanations and justifications. Status posters, student work analysis, reengagement lessons, peer editing and review, are just some of the instructional techniques we emphasized in our mathematics workshops.

What TRU has allowed us to do is to frame individual issues like tracking as part of the larger picture. Tracking is now framed as an issue of access, and potential remedies point to the domain of Agency, Ownership, and Identity (Dimension 4). That is, we now know we need to do more than just give students access (a partial “solution” to the issue of equity), and to do so in ways that allow students to see themselves as mathematical thinkers and problem solvers. TRU has helped to frame our professional development in more coherent ways.

**Productive uses of assessment**

Perhaps counterintuitively, richer and deeper mathematical tasks (especially those amenable to multiple approaches and/or using multiple representations) provide
greater access to important mathematics and support rich classroom discourse (and thus possibilities for greater agency). To counter more than a decade of California’s emphasis on skills-oriented high stakes tests, starting in 1997, SVMI formed the Mathematics Assessment Collaborative (MAC). We made a strategic decision to invest in a mathematics performance assessment test that assessed higher cognitive levels, that needed to be hand-scored, and that produced rich examples of student work. We commissioned the MAC/Mathematics Assessment Resource Service (MARS) performance assessment tests in 1999. These assessments, used for both summative and formative purposes in classrooms, provide rich mathematical content, generate student experiences with high levels of cognitive demand, assess students’ ability to be productive in high cognitive demand situations, and develop teachers’ skills in implementing formative assessment practices. Three dimensions of TRU Math – Mathematical Content, Cognitive Demand, and Formative Assessment - provide the framework for our deep professional learning using the MAC/MARS Performance Assessment tasks. Our collective debriefs on student work are now framed in the language of TRU. The mathematics of tasks used in the performance assessments is designed along a learning progression, where the initial questions are accessible to nearly all students and they can demonstrate what they know. Additional questions probe whether students can demonstrate that they are meeting grade/course level standards. The tasks assess student thinking at higher cognitive levels to measure conceptual understanding, applications, generalizations and/or justifications. Our professional development, using performance assessments, strengthens teacher knowledge and focuses on both the mathematical content being taught and the levels of cognition in which the students are engaged. Teachers learn that a mathematically powerful program includes a balanced diet of the levels in Norman Webb’s Depth of Knowledge approach (Webb, 2007) or the levels of Cognitive Demand characterized by Smith and Stein (2011).

In addition to the MARS performance tasks, SVMI often engages educators in the Formative Assessment Lessons (FALs). These lessons, produced by the same team that produced the TRU framework, support all five dimensions of TRU in the classroom. Educators experience the lessons as learners. Finally, TRU Dimension 5 (Formative Assessment) guides SVMI’s essential work to elicit student thinking and use that thinking to promote further learning. Teachers select a MARS performance task focused on the mathematics content of the unit they are teaching. During the unit, the teachers administer the task to their classes. Collectively, teachers score and analyze their students’ work, identifying common errors, misconceptions, reasoning flaws, varied approaches and representations, successful explanations, and other artifacts of student thinking. The teachers then use actual student work samples to design a lesson, called a reengagement lesson. The lesson is taught by presenting these mined student gems to pose learning disequilibrium or cognitive conflict. Students are asked to critique,
analyze, or explain one another’s thinking, arriving at correct solutions, reasoning about varied approaches, or improving mathematical explanations or justifications.

Reflections

TRU Mathematics has changed our thinking in several ways. Prior to TRU, we would awkwardly attempt to describe the role of the student in the classroom. To address the varied aspects of their role we would discuss the classroom environment and the culture that needed to be established. We discussed the student role in group work and aspects of status, accountability, inter-personal skills, and self-reliance. Then we would attempt to address classroom discourse, including different talk moves, good questioning strategies such as funnelling versus focus questions, strong explanations and justifications, and students’ perseverance. Then we would focus on academic and mathematical language, with special treatment for students whose first language is not English. This professional development “to-do” list got longer and longer as more research, such as growth mindset and students’ disposition toward learning mathematics came to light. When TRU Math introduced the dimension of Agency, Authority and Identity, lights came on for us. All the descriptions and discrete categorization, formerly described, are captured and condensed into targeted work to develop student with agency, authority and identity. TRU provided both concise language and the needed focus on the core essence of the student role in learning mathematics. This was enlightening and liberating.

At the same time, we still face significant challenges. One is to help build self-sustaining Teacher Learning Communities. It’s still an open question as to how to guarantee the longevity and purposefulness of Teacher Learning Communities (TLCs), and how to make TRU so natural a part of a TLC’s work that it automatically frames issues through the lens of TRU. A second is how to secure administrative buy-in at both the school and district levels. It is easy for an administrator to undermine the work of a TLC by, for example, mandating skills testing, not providing adequate time or resources for the TLC to work effectively, or trying to implement so many “helpful” initiatives that coherence is lost. We have begun working on tools that support administrators in supporting TLCs.

TRU IN CHICAGO: SUPPORTING SYSTEMIC CHANGE

(RUTH HAUMERSEN, ALANNA MERTENS, AND LYNN NARASIMHAN, WITH NICOLE LOUIE)

We are writing as a group of mathematics educators who have come together to look at TRU more deeply as it has come into the work of the Chicago P12 (Pre-kindergarten through Grade 12) Mathematics Collaborative. The Collaborative began in 2012 as a partnership between the Department of Science Technology,
Engineering and Mathematics (STEM) of the Chicago Public Schools (CPS) and local institutions of higher education, with the goal of strengthening instructional practice and increasing student success in mathematics. We created and implemented a mathematics professional learning model with a district-wide reach (some 1300 teachers across 500 schools) and additional support for a subset (110) of the district’s schools. The model included cycles of teacher workshops, administrator institutes, cross-site professional learning communities for teachers and school teams, and both individual and collaborative coaching.

TRU entered the Collaborative’s work in 2014. When we were first introduced to the framework at a meeting hosted by the Silicon Valley Mathematics Initiative, we saw great potential for the five dimensions to powerfully summarize the shifts in instruction that we had been working toward in our professional learning model, and for the framework to provide a common language for teachers and administrators to have meaningful conversations around high-quality mathematics instruction across a very large and diverse district. We, therefore, began to ground all professional learning in mathematics in the TRU framework. At each district-wide teacher and administrator session, we looked deeply at one dimension of TRU and explored its significance through participants’ own engagement in rich mathematical experiences, as well as through observation of and reflection on how the dimension might look in CPS classrooms. Time was also given to consider next steps and to do some collaborative planning in light of that session’s highlighted TRU dimension. We placed a significant emphasis on the use of the TRU Conversation Guide, especially the core questions and “Think About” sections, in all mathematics professional learning, but particularly in the professional learning community sessions and during collaborative coaching.

Since we began our work with TRU, we have seen remarkable changes in how teachers talk about their vision for mathematics instruction. One teacher reflected, “I used to think that [this work] was beyond the capabilities of not only my teaching practice but also my students’ ability level. Now I think that … I am more than capable and my students are more than able.” We have also seen shifts in how teachers relate to one another. As another teacher described, “I feel that teachers are becoming more open about sharing practices as a result of these trainings…. Observing other teachers’ practices has been the most valuable to me.”

TRU, and the Conversation Guide in particular, has had a major role in facilitating these shifts. To be clear, it is not a silver bullet that has solved all of our problems. But it has supported us to enact – and learn about enacting – a vision of teacher learning that emphasizes (1) a robust vision of powerful mathematics instruction, (2) building social capital alongside individual human capital, and (3) teacher agency, authority, and professionalism. We discuss each of these points below.
A Robust Vision of Powerful Mathematics Instruction

When the Collaborative was launched, we focused on instructional strategies that had great potential to increase students’ opportunities to make sense of big mathematical ideas and explain and justify their thinking—that is, to support the vision of rich, powerful mathematics instruction that we had. These strategies included Math Talks (http://www.sfusdmath.org/math-talks-resources.html), Three Reads (http://www.sfusdmath.org/3-read-protocol.html), and Formative Assessment Lessons (http://map.mathshell.org/lessons.php). However, we found that teachers often focused on the “what” of the strategies instead of the “why.” Instead of reasoning about their instructional decisions in terms of rich, powerful goals, they were concerned with following protocols and ticking checkboxes.

TRU helped us make the vision in our heads more explicit for teachers, and it helped us develop a shared language that everyone in the Collaborative could use to articulate and reinforce their goals for their instruction. When teachers ask whether the protocol says students should have 2 minutes or 5 minutes of independent think time, TRU helps us bring them back to a vision of powerful mathematics instruction with questions like, “How do you think that would affect students’ access to the mathematics in this task?” Teachers themselves—including some who have never attended a Collaborative workshop but have colleagues at their schools who have—also bring up these kinds of questions during planning meetings and peer observations.

We knew that it was important for teachers to connect instructional strategies to bigger ideas, goals, and principles—the same way it is important for students to connect procedures and algorithms to underlying concepts. What TRU did was show us how powerful it could be to support those connections with a framework and language that teachers could return to again and again, developing personal meanings and connections that further fueled Collaborative work.

Building Social Capital

Early in the project, our focus was on improving individual teachers’ practice. We worked to develop teachers’ human capital, deepening their content knowledge and pedagogical content knowledge and providing classroom resources and instructional strategies. In early professional learning community meetings, however, we began to see the value of bringing teachers together in a safe environment where they could share problems of practice and successful classroom experiences. Teachers who initially said things like “I can’t do this in my classroom with my students” were trying new strategies after hearing success stories from colleagues at neighboring schools.

As the project evolved, we shifted our focus to building social capital—trust and collaboration between teachers—through increasing opportunities for teachers to make their practice public within and across schools (Leana, 2011). This shift
towards increased collaboration and public practice was reinforced as we began to see that the TRU framework not only describes dimensions of powerful learning for students, but also dimensions of powerful professional learning for adults (Schoenfeld, 2015). Thus, the use of the TRU framework at the scale of professional learning for teachers provides consistency: teachers who are focused on creating powerful learning environments for their students are experiencing such environments in their own learning. It also supports the idea that to be effective, learning environments – whether for teachers or students – need to be shared and collaborative in nature.

As this shift took hold, we saw teachers engaging with one another around problems of practice related to the vision set forth in TRU. One principal described the transformation she was seeing: “Our whole staff is coming kind of to a threshold where they’re becoming a collaborative staff. They are trusting each other, to take [one another’s] criticism and also to do something positive with it.”

**Teacher Agency, Authority, and Professionalism**

In tandem with working to support strong teacher communities and a strong instructional vision, we have learned to support teachers’ sense of agency, authority, and professionalism, in parallel to asking them to support students’ agency, authority, and identity. Just as there are many “right” ways to solve rich mathematics problems, there are many “right” ways to teach for robust understanding of mathematics, and teachers have clearly communicated that they appreciate having space to take on leadership in developing their own practice – without being told what to do, and without being evaluated.

When we started using TRU, we saw possibilities for it to empower teachers. As we were introducing instructional strategies, TRU provided room for teachers to continue to use their own strategies or modify ours and connect them to the Collaborative via the larger goals articulated in the dimensions. The questions in the Conversation Guide also provided opportunities for teachers to engage in deeper thinking, as they made their own sense of the dimensions, connections between the dimensions and their current practice, and ways they wanted to improve or grow. The core questions in the Conversation Guide also created a safety net for teachers engaging in peer observations. Because they are open-ended with no right or wrong answers, it became easier for every teacher to participate in discussing them. Additionally, they made focusing on a particular area less threatening, especially when teachers themselves had picked the focal question for the day. Instead of picking apart an individual teacher’s practice because it fell short on a rubric or checklist, we could think together about a question the lead teacher had shared to develop not only her practice but our collective practice as teachers of mathematics.

In practice, teachers took on responsibility not only for trying new strategies and analyzing their effects on student learning but also for organizing and sustaining
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collegial collaboration at their schools. TRU has helped them to develop a shared focus that every team member could find a personal stake in, and to which everyone had something to contribute. At one school, a teacher described this process as creating “a system ... to not check what people are doing, but to get ourselves into each other’s lives, our teaching lives. So we started meeting together as a mathematics team to figure out how can we set up a schedule so we can get into each other’s classrooms.” The mathematics team, which consisted of teachers from pre-Kindergarten to middle school, used the TRU dimensions to sharpen the focus of these observations and created a document to support the peer collaboration process. A team member said that the purpose of the document was to help give:

a focus to the discussion. And to have that same conversation happening across [the] entire school ... There’s power in being able to ask the same questions, look for the same things, talk about how is this giving Agency and Authority to the students while we’re doing a lesson.

A teacher outside of the mathematics team who began to take part in these peer observations reflected that:

after you do it a couple times, I think it becomes much easier to ... realize that ... your colleagues are simply there to help ... it opens up, it’s funny, because it kind of opens up the same door that we want for the kids. The comfort level is there so we can really share where we’re at with [instructional strategies].

It is through this kind of shared ownership and trust that teachers become empowered to grow their own practice using the common language and vision provided by the TRU Dimensions as they work to create mathematically powerful classrooms for their students.

Concluding Remarks

Although we have treated them as separate, a robust vision for instruction, building social capital, and teacher agency, authority, and professionalism are deeply intertwined. To build social capital, we have leveraged our instructional vision as an organizing tool, as have teachers. And teacher agency, authority, and professionalism both stem from and contribute to strong social capital and a vision that is clear and coherent without being prescriptive. At a time when teachers are constantly bombarded with resources and strategies, and are faced with constant pressure to raise student achievement, we suggest that less is more. Having fewer tools can be immensely generative, when those tools are open-ended enough to support teachers to make their own sense of them, take ownership, work with others to solve problems of practice, and promote a collaborative culture of transparency, reflection, and growth.
We are a research-practice partnership between two teacher-leadership organizations, Math for America (MfA) and the New York State Master Teacher Program (NYSMTP), and two universities, Montclair State University (MSU) and the State University of New York (SUNY), Buffalo State. MfA is a nonprofit organization based in NYC with a mission to improve mathematics instruction in the United States. To accomplish this mission, MfA works to retain talented and experienced teachers through selective, four-year fellowships that provide ongoing professional and leadership opportunities. NYSMTP is an independent, publicly funded program explicitly based on MfA’s model aimed at improving STEM teacher retention across the state of New York. Fellowships at MfA and NYSMTP provide teachers opportunities to work with one another, outside of school hours, to pursue a variety of self-selected learning opportunities. These opportunities fall into different categories and include teacher-led learning teams dedicated to understanding how to use high-quality instructional materials effectively.

An example of the type of high-quality instructional materials teachers explore in both programs is the Classroom Challenges Formative Assessment Lesson (FAL) collection. Classroom Challenges, often simply called FALs\(^5\), are a set of 100 free mathematics lessons developed by a team at the University of California at Berkeley and the Shell Centre for Mathematics Education at the University of Nottingham. The lessons support teachers’ formative assessment in important mathematical ideas and practices articulated in the CCSSM.

Our partnership between teacher-leadership programs and universities is creating a repository of video cases based on FALs taught by teachers at MfA and NYSMTP in a diverse set of classrooms across New York State. The video cases are intended as objects of study for communities of practice (Wenger, 1998) dedicated to understanding how to teach FALs. The communities of practice we support foster interactions within and across populations of prospective, early career, and practising teachers; this later group includes teachers who have been awarded Master Teacher fellowships by MfA and NYSMTP, as well as others such as colleagues in their schools.

Each case includes a video segment of secondary instruction of an FAL taught by an MfA or NYSMTP fellow along with supporting materials that provide context. Each case also includes a set of discussion prompts, based on the TRU Framework, that support teachers, coaches, professional development leaders and teacher educators in facilitating discussions about mathematics teaching and learning.

The video cases are not intended as exemplars. Rather we understand them within the communities of practice framework as objects of study that make it possible to develop collective knowledge about how to use FALs effectively in different contexts. Of particular interest are emerging, collective understandings about how
students understand specific mathematical ideas (e.g. sample spaces, domain and range) and teaching moves that can support student thinking in these areas.

As the video cases are used in different contexts (professional learning communities, methods courses, etc.) our team collects commentary (e.g., discussions, mathematical solutions) on the case and adds these artifacts to the materials. This commentary supports the shared repertoire that allows different members of the community to deepen their understanding of the teaching and learning generally and as it pertains to specific lessons.

From the beginning of our research, all partners believed it was important to situate the case study materials in the context of a research-based framework characterizing the dimensions of high quality instruction. We chose TRU because of its accessibility, comprehensiveness, readability, and abundance of open-source support materials.

In what follows we will discuss two challenges we faced related to TRU in developing and using the video cases. The first is deciding how to select video clips that can support rich discussions for teachers in different contexts and at different stages of their careers. The second is which TRU tools we should include with the case materials to foster ideas that can be used collectively not only to improve the teaching of specific lessons, but to deepen our understandings of practice generally.

Theoretical and practical challenges

There is a gap in research about how the same records of teaching, such as those in our video case studies, can be used effectively for learning in different contexts (Ball, Ben-Perez, & Cohen, 2014). This relates to our project particularly with respect to the video clips we select that are essential ingredients in the video cases. We have approached this issue by using the TRU Framework to guide our selection of video, and studying the ensuing discussions as teachers mutually engage in analyzing and discussing the video, and iteratively improving our selection over time. We see the TRU Framework, the rationale for selecting video, and the video itself, all as integral in developing the shared repertoire that sustains our communities of practice in a joint enterprise of learning how to teach FALs.

Our initial thinking was that we could start with a particular dimension of TRU, for example *Agency, Ownership, and Identity*, and look for video that we believed might support rich discussions in that area. We imagined that for a particular FAL, we might end up with several video cases each centered on a particular dimension of TRU. We found, however, that video clips selected in this way would often support some users but not others. A certain clip selected that supported a rich discussion for a group of prospective teachers, for example, might fall flat with in-service teachers. Over time we learned that the heart of this difficulty related to *The Mathematics*. Specifically, if the clip did not allow for exploration of a rich mathematical activity (either because aspects of the activity weren’t clear on tape,
or because the lesson activity wasn’t particularly rich), the clip would not work across communities. This changed our approach, and we now use *The Mathematics* as a starting point for selecting the clips and engaging in the video case materials. This approach is theoretically consistent with the nonlinear representation of TRU that places The Mathematics at the center (Figure 2).

![TRU Diagram](image)

Figure 2. A non-linear representation of TRU, representing the interconnections of the five TRU dimensions, with mathematics at the core (Schoenfeld, 2016, with permission)

Another challenge we faced was in deciding which TRU tools to use when discussing the video. As in selecting the video, we proceeded by iteratively testing different approaches. We used tools individually and in combination, pulling from the TRU Conversation Guide (Baldinger & Louie, 2014), the Observation Guide (Schoenfeld and the Teaching for Robust Understanding Project, 2016), *On Target* (Schoenfeld and the Teaching for Robust Understanding Project, 2018), and the Framework itself. We found that each of these tools appeared to produce meaningful learning opportunities for practising teachers and in prospective teacher classrooms. We were particularly impressed with the learning opportunities afforded by *On Target*; practising teachers were able to characterize and unpack complex teaching situations by locating teaching episodes on a target with various descriptors corresponding to a particular dimension. They found using the tool particularly helpful for their individual growth, often remarking that they knew exactly where they fell on the targets in their own instruction. Watching video of others and thinking about how to move closer to the bullseye gave them ideas for their own practice. This tool also successfully scaffolds discussion of complex moments for prospective teachers; the descriptors, along with the Framework, gave them an accessible language to notice what matters.

And yet something was missing in terms of our project’s overall goals. We use communities of practice as a way to both describe and characterize the learning
process we are studying. The case studies and TRU-based discussions taking place as various teacher groups engage with the materials allow us to capture the development of teachers’ thinking about mathematics instruction, and one of our aims is to reify teachers’ ideas about the video in ways that establish a “community memory” (Orr, 1990) about teaching and learning FALs. With On Target, as with other tools, the great flexibility in the tools led to considerable variation in the analysis by different groups, and often failed to unearth common themes that could be refined and built on over time.

We made progress in this area by simplifying our approach somewhat and adapting one of the most basic TRU tools: a description of the Framework written from the students’ point of view in the Observation Guide. Here the dimensions are framed as questions – for example The Mathematics is introduced with the question: “What’s the big mathematical idea in this lesson? How does it connect to what I already know?”

As we had already decided to select clips based on The Mathematics, we wrote the following questions that are now answered by users of the video cases after doing the task in the lesson and before they watch the video:

1. What are different ways to solve the problem?
2. How do the different ways to solve the problem illuminate the big mathematical idea(s) in this lesson?
3. How do different ways to solve the problem connect to one another?
4. What approaches are students likely to take when trying to solve the problem?

For the videos, we created questions for each dimension, again based on the school students’ point of view. For Agency, Ownership, and Identity we used the characterization: “Do I get to explain, to present my ideas? Are they built on? Am I recognized as being capable and able to contribute in meaningful ways?” to write the following questions:

1. What do the students’ different explanations tell us about how they might be thinking and what they might understand?
2. Imagine we could go back in time to this part of the lesson and put ourselves in the teacher’s shoes. What questions might we ask or what moves might we make to build on the students’ thinking?

While these questions are fewer than those posed in On Target, we have found that they still support rich discussions around teaching and learning. Critically to our project’s goals, these questions have led to shared noticings across groups that focus on students’ mathematical understandings. As different conversations are recorded, summarized, and built upon, we see this as reification of our evolving and collective understanding about to teach for robust understanding.
Discussion

Our intent is to produce video cases that give prospective and early career teachers the opportunity to think about complex situations in the classroom and put themselves in the position of decision maker. We hope that focusing on teaching practices rather than the teacher will allow these viewers to consider teachable moments, what one might do next in a lesson, how to handle particular events during the lesson, and the discourse in the classroom. These considerations will build understanding of teaching practices. For practising teachers the videos provide opportunities for discussion across the five dimensions that build on their own experiences and help foster a vision of classrooms that feature mathematically rich, accessible, and cognitively demanding learning environments.

The TRU Framework is supported by an impressive array of high-quality tools. In our experience all the tools support the learning experiences described above, and some have worked better than others to develop a shared repertoire of practices and orientations across pre-service and in-service environments. Our observations that support this position, are still based on a relatively small number of users. Our next step is to build and maintain a micro-site for the cases. This micro-site will be open to any individual or group of educators, and we are planning to reflect on feedback from the users for continued refinement, revision, and development.

TRU AND LESSON STUDY

(ANGELA DOSALMAS, HEATHER FINK, SANDRA RUIZ, ALYSSA SAYAVEDRA, ALAN SCHOENFELD, ANNA WELTMAN, AND ANNA ZARKH; SUZANNE DONOVAN AND KAREN TRAN; COURTNEY ORTEGA AND MARY REED; CATHERINE LEWIS)

The TRU-Lesson Study partnership (UC Berkeley, the Strategic Education Research Partnership (SERP) Institute, Mills College, and the Oakland Unified School District) is a National Science Foundation-funded effort to enhance teacher professional development in ways that combine the strengths of the TRU framework and Lesson Study, a collaborative teacher learning program with origins in Japan. TRU-Lesson Study, like Lesson Study, engages teachers in inquiry and reflection around important problems of practice through inquiry cycles of studying, planning, enacting, and reflecting that culminate in live research lessons (Lewis & Hurd, 2011). Every step of the TRU-LS inquiry cycle, from design to enactment to reflection, is framed by the vision of mathematics teaching and learning provided by the TRU Framework. Hence, Lesson Study provides the overarching activity structure while TRU provides a theoretical and structural frame for professional learning content (Schoenfeld, Dosalmas, Fink, Sayavedra, Weltman, Zarkh, & Zuniga-Ruiz, 2019).

Ultimately, the goal of TRU-related professional development is for individual teachers and teacher learning communities to “own” and to internalize TRU – for
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the principles underlying TRU to frame both lesson planning and in-the-moment enactment of lessons, as mechanisms to support powerful instruction. This raises significant tensions. On the one hand there is a body of knowledge to be internalized (not simply “learned” – the goal is not to talk about or recognize TRU dimensions, but to think with them); on the other hand, there are issues of building and teacher autonomy to be respected.

As planners and facilitators of professional development, we often found ourselves wondering: How do we as teacher educators set learning goals that inspire but do not constrain or impose? How do we present teachers with a framework meant to structure their work, but also support dialogue that allows teachers to engage with issues that are meaningful for them and their community? How do we avoid making “learning TRU” the goal, rather than “learning to use TRU as a means to inquire into one’s practice, foster communication, and improve instruction”? Such issues arise in all professional development, of course, especially in work that tries to be respectful of and support teachers’ professionalism. We were not always successful in negotiating these tensions; some sessions were too much about TRU and some seemed to push our agenda more than might have been helpful. But over time, with feedback and review after every session with teachers, we learned how to better integrate the TRU Framework into our teachers’ inquiry projects in ways that provided teachers with an ambitious horizon of mathematics teaching and learning to aim for, but also nurtured teachers’ own valued problems of practice and ownership over them as a community.

In the remainder of this section we highlight two decisions that we faced as teacher educators designing and implementing TRU-Lesson Study in which the tension between structure and agency was salient.

Decision 1: How should teacher educators integrate the TRU Framework into the lesson study activity structure?

TRU can be used in generative fashion as well as for reflection. However, there is a lot to learn before one can be fluent with it. An early issue, then, was how we might integrate TRU with the more classic lesson study processes. Lesson study provides an activity structure within which teachers can pursue collaborative inquiry projects while the TRU Framework, on the other hand, comes with no such structure for activity.

Initially, we used TRU as a lens for reflecting on videos of practice. For a particular video, what could we say about (for example) students’ agency? What opportunities did the students have to develop productive mathematical identities, and how might the space of opportunities be opened up? Similarly, when we integrated TRU and Lesson Study, TRU played a natural role in the formal lesson commentaries – How rich was the mathematics; when and where were the students engaged in productive struggle; which students participated, in which ways; what opportunities were there for agency, etc.
As intended, the TRU Framework supported teachers in noticing features of lessons, students’ engagement, and teachers’ decisions as experienced through the eyes of a student. However, using the TRU Framework primarily as a reflection tool had some drawbacks related to the balance between agency and structure. We began to see some teachers understanding the framework as a static, canonical entity to be used as a reference for checking whether the lesson they observed or planned had all of the features “required” by TRU. They used the framework to label their observations, with the result that their reflections tended to stay in the territory of what was noticed or planned, rather than how what was noticed or planned came about or why it mattered for students. Using the TRU Framework often became the endpoint of conversations—an evaluation rather than the beginning of deeper reflection. This led us to develop a new activity structure aimed at enhancing teachers’ own agendas.

The new activity structure, the TRU Inquiry Cycle, was incorporated into the study phase of Lesson Study. As in Lesson Study, teachers began by setting a shared goal based in a current problem of practice. Then, to explore that goal, they choose a pedagogical strategy to try in their classrooms. The teachers collected classroom artifacts demonstrating how students reacted to the pedagogical strategy, brought them to department meetings, and reflected on how the strategies had played out, using the TRU Framework. Typically, those reflections spurred revisions of their statements of their problem of practice and goals; those, in turn, led to the selection of new or revised pedagogical strategies, launching another round of the inquiry cycle. As the teachers came to use the TRU Framework to pose questions for inquiry and to design teaching experiments, they were positioned as initiators of the work rather than as consumers. This gave them authority over the meaning of the framework, with the TRU team playing a supportive role rather than dictating meaning from a position of authority.

**Decision 2: How should mathematics teacher educators reference and leverage the TRU Framework in conversation with teachers?**

A second decision that we faced in balancing the structure provided by the TRU Framework with our commitment to teachers having agency over what and how they learned concerned in-the-moment decisions of when and how to incorporate the TRU Framework into teacher discussions. Within a given conversation, we asked ourselves: when was a good time to push discussions in particular direction, and how should such moves be best articulated? We have found that these in-the-moment decisions cue facilitators’ and TRU’s positioning in ways that influence teachers’ sense of agency, authority and identity. Here an analogy to classroom instruction may be useful. The classic “demonstrate and practice” form of instruction in mathematics (Lappan & Phillips, 2009) has the virtue of clarity: students know what they are supposed to be doing. However, it denies them agency: they are doing “other people’s mathematics.” In contrast,
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problem-based learning starts with issues (admittedly, typically chosen by instructors) and then builds on student thinking. Our work takes this approach one step further. We find that it is best to start with goals and problem statements that come from teachers. Then, appropriately timed interventions using TRU as a tool can help demonstrate its value, in service of the teachers’ goals. To give one example, teachers at a particular site were concerned that their students did not persevere when working on the problems the teachers had designed. They had created resources for the students; why weren’t the students using them?

The challenge with regard to framing things in terms of perseverance is that it can be a dead end: what can we do if the students won't persevere? When this issue arose, the TRU facilitators helped re-frame the question. Perseverance is a function of agency (Dimension 4 of TRU): students are likely to persevere if they think they have a chance of success. How do they develop that sense of agency? By being successful. How does that happen? When instruction offers students challenges that are within reach (a matter of formative assessment and cognitive demand, Dimensions 5 and 2), giving students an opportunity to make legitimate progress and build agency.

This kind of re-framing helped teachers pursue their own goals (“We need to craft tasks and lessons in which students can experience legitimate success”), both supporting teacher agency and demonstrating the ways in which TRU can facilitate their own agendas. We have found that if communities of teachers develop routines around using the TRU Framework to problematize their inquiry goals, and around actively negotiating how the TRU Framework can support their inquiry, that in-the-moment integrations of the TRU Framework into conversation are more likely to be taken up as inviting, rather than constraining, teachers’ agency. However, we also found that each decision to bring the TRU Framework into a teacher conversation already-in-progress must be carefully considered. What impact will the TRU Framework have on the trajectory of conversation? How will its introduction position the teachers and facilitators with respect to each other and the broader professional community? What shared meanings have teachers and facilitators begun to develop around the TRU Dimensions, and how will invoking the dimensions constrain or open up dialogue about those meanings? Any introduction of the framework will necessarily redirect conversation, reposition teachers and facilitators, and assert meanings for key terms.

Discussion

The non-prescriptive nature of the TRU Framework – TRU does not tell teachers or teacher educators what to do – is a significant virtue, but it may also be its greatest challenge. We have found ourselves struggling to balance the need to bring TRU forward when we see it can help and the need to respect teacher agency and authority. There are, we suspect, no easy solutions to this dilemma – although we
hope that the construction of additional tools will provide more resources for teacher educators as they deal with this challenge.

CONCLUDING DISCUSSION

The TRU framework was designed to focus on what counts – to the degree that a framework with five dimensions can focus. Any distillation of a phenomenon as complex as teaching into five dimensions necessarily foregrounds some critical concerns and backgrounds others, issues of race and power being examples raised in section 2. It’s not that such issues are not implicated; one can’t reasonably consider issues of equitable access and agency/ownership/identity without dealing with issues of race and power head on. But, there’s a lot of unpacking to be done to help TRU deal adequately with such issues, and to provide useful tools for addressing them. Doing so with any degree of success will require a significant program of research and development. Learning how to bring such concerns naturally into TRU-based professional development will take some learning on the part of teacher educators.

The non-prescriptive character of TRU provides essential opportunities and in doing so raises a set of challenges and tensions. Ultimately, there is a need for powerful and self-sustaining teacher learning communities – communities that continue to refine their members’ understandings and practices in ongoing ways. This is essential for two reasons: (1) learning communities are the best “growth medium” in which understanding can take hold and grow, and (2) as a matter of scale, there simply aren’t enough teacher educators to provide the relevant support for teachers, on their own. Teacher educators can serve as catalysts, but part of our role needs to be to help the communities we help foster become increasingly independent (but supported with good tools, of course.)

In the US, at least, this means that community building is a critical part of the challenge – Lortie’s (1975) invocation of the “egg crate” to describe teacher isolation is still a significant reality. Bringing teachers together and telling them what to do is deprofessionalizing – and it doesn’t work. As a matter of respect and because meaningful attention to teacher support is context-dependent, there must be substantial flexibility. But with such flexibility come tensions related to focus and coherence. We have found that there are certain patterns of teachers’ perceived needs: in the US, Dimension 4 (agency, ownership, and identity) is often perceived as a needed expansion of a focus on equitable access (Dimension 3), and a good place to dig in at first; after some time with Dimension 4, it becomes clear that efforts will be more effective if one understands how to help students engage in productive struggle (Dimension 2), and TLCs often turn to that. This, of course, necessitates attention to student thinking (Dimension 5) – all the time, with content worth engaging with (Dimension 1).

What we have just outlined is one possible order of a curriculum for professional development. Its effectiveness would depend, of course, on community; on issues
being meaningful and workable for participants; on their making it their own. If that sounds familiar, it should. TRU is a theory of productive learning environments, and if teacher educators are to help teachers shape powerful learning communities, those communities themselves should do well along the dimensions of TRU (see Schoenfeld 2015 for more detail).

There is one further challenge to community building. Point (2) above was that even if teacher educators in the US were familiar with and predisposed toward using TRU, there is not an adequate number of teacher educators to provide the relevant support. Thus, further work needs to be done along at least two dimensions: helping communities that have made significant progress to become self-sustaining, so that teacher educators can be freed to have broader impact, and building networks in ways that teachers themselves can become ambassadors of change, and mentors to other teachers.

Making this happen is a significant challenge. But as the discussions in this chapter indicate, taking on that challenge is a source of significant learning for both teacher educators and the teachers they work with.

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NOTES

1 See https://truframework.org/tru-conversation-guide/
2 In California, most teachers earn their Teaching Credential subsequent to earning their undergraduate (Bachelor’s) degree. The MACSME program combines this professional credential with an academic program that leads to the Master’s degree.
3 See https://docs.google.com/document/d/1NtkzSgLoLReU-nG6TTedvLfrSAKm7VT2GGwipBwkJk/edit
5 See also the use of FALs in Section 4.3.
6 A TRU tool we created offered a list of strategies. Teachers were not constrained to this list, but they typically used it as a resource and selected strategies from it.