Teaching for Robust Understanding with Lesson Study

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Abstract This chapter describes the synthesis of two powerful approaches to professional development, the Teaching for Robust Understanding (TRU) framework and Lesson Study. The synthesis is known as TRU-Lesson Study.

The TRU framework identifies five essential dimensions of classroom practice: (1) the Content; (2) Cognitive Demand; (3) Equitable Access; (4) Agency, Ownership, and Identity; and (5) Formative Assessment. When classroom practices engage students meaningfully along these five dimensions, students become knowledgeable and resourceful thinkers and problem solvers. In TRU-based professional development, teacher learning communities negotiate their visions of teaching and learning collaboratively by reflecting on artifacts of practice using the TRU framework.

In math-focused Lesson Study (LS), teachers work together to design, teach, and reflect on a lesson that focuses on key mathematical issues and students' engagement with them. Widespread in Japan, Lesson Study is a powerful mechanism for building and sharing understandings of mathematics content, teaching, and learning.

Teachers in the USA typically have little collective time to reflect on teaching practice. TRU-Lesson Study (TRU-LS) supports the growth of teacher learning communities and their engagement with key ideas and practices of TRU and Lesson Study. Like Lesson Study, it profits from teachers' concerted attention to lesson design and reflection on the hypotheses reflected in the design. Like TRU-based

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professional development, it supports teachers to work together explicitly on key dimensions of classroom practice. This paper describes TRU-Lesson Study and provides descriptions of how it plays out in practice.

Keywords Lesson Study · Teaching for Robust Understanding · TRU framework · Professional development

1 Background and Context

1.1 Introduction and Overview

As this volume indicates, Lesson Study (LS) is a worldwide mechanism for professional development. Yet, just as teaching varies from country to country (Stigler and Hiebert 1999), so does Lesson Study, based on the cultural affordances available to teachers (Chen 2017, Perry and Lewis 2009). In the US, teachers typically have little collective time for collaboration; they do not have national curricula supporting rich mathematics instruction; and there has not been a historical tradition of attending to student thinking as students engage with rich mathematics. For Lesson Study to take root and thrive in the US, it must address these realities.

Within mathematics education, Lesson Study's power comes in large part from teachers' collaborative efforts to understand the richness and complexity of mathematical ideas and practices and to support students in engaging with them. When teachers design and reflect on lessons that focus on rich content and student learning of it, their collective negotiations of problems of practice provide an ongoing mechanism to share and develop pedagogical understandings.

The Teaching for Robust Understanding (TRU) framework (Schoenfeld 2013, 2014, 2015, 2017) describes what matters for equitable and robust learning in mathematics classrooms. As elaborated in Sect. 1.3, the TRU framework focuses on five key dimensions of learning environments: (1) the Content (here mathematics); (2) Cognitive Demand; (3) Equitable Access; (4) Agency, Ownership, and Identity; and (5) Formative Assessment. The first dimension concerns the richness and complexity of mathematical ideas and practices essential for powerful mathematics learning. The remaining four dimensions focus on students' experiences of the mathematics and the impacts of those experiences on their mathematical development and understanding.

TRU's model of professional learning engages teacher learning communities (TLCs) in ongoing inquiry cycles grounded in the TRU framework. Rather than specifying any particular approach to teaching, TRU focuses teachers' attention on attending explicitly to aspects of practice that are essential for supporting students' individual and collaborative engagement with rich mathematical ideas and practices. In regularly scheduled meetings, teachers share artifacts from their classrooms that capture evidence of focal problems of practice and are supported to use the TRU framework to inquire more deeply into key practices. Between meetings teachers try new strategies, visit each other's classrooms, and/or collect classroom artifacts to bring to the next meeting.

There is a strong philosophical affinity between TRU and LS. Both attend explicitly to how students experience instruction; both see discourse within TLCs as a venue for teachers' ongoing professional growth; and both value teacher professionalism. In Japan, LS is well established; so are teacher collaboration, peer support for collective growth, and teacher professionalism. In the USA, collaborative support for teacher growth is rare (Horn et al. 2017; Little 2002). Lortie (1975) characterized school buildings as "egg crates," in which each teacher works in his or her room entirely isolated from colleagues. Today, this is still often the case. TLCs in the USA struggle to develop the trust and depth of conversation necessary for inquiring into complex problems of practice in ways that could lead to learning (Horn and Little, 2010; Horn et al. 2017) – a significant obstacle to the implementation of Lesson Study. In addition, teachers in the USA are not necessarily as well prepared as Japanese teachers to focus in detail on student thinking. TRU-LS is designed to help build teacher communities and to focus on student thinking in ways that allow the practices of Lesson Study to take hold, enriching what teachers can learn from focused reflection on designing instruction and student engagement with important mathematical ideas.

1.2 Lesson Study Model[1]

Lesson Study has established a foothold in the USA over the past 15 years (Akiba et al. 2014; Hill 2011). Not surprisingly, there is a range of LS adaptations to the US context. TRU-LS draws on the theoretical model of Lesson Study proposed by Lewis and colleagues, aimed at adapting Lesson Study to the US context (Lewis et al. 2009; Lewis et al. 2006; Perry and Lewis 2009; Takahashi and McDougal 2016; Watanabe et al. 2008). The core aspects of one LS cycle are shown in Fig. 1.

In one LS cycle, a TLC moves together through the four-phase process in Fig. 1. They begin by studying curriculum and standards as well as considering their long-term goals for student learning. In this phase the TLC decides on a research theme and theory of action and selects a mathematical topic on which to focus. It conducts background research, looking at what is known about student understanding of the topic and at relevant instructional materials (possibly including exemplary lessons). With this refined understanding, they *plan* a lesson, identifying precise issues that members of the team want to focus on when the lesson is taught. One or more teachers then *conduct* the research lesson, while the remaining teachers and invited educators observe, gathering evidence of student thinking. Participants then *reflect* on how the lesson played out and discuss their observations with an outside expert. They then move on to their next LS cycle.

[1]Our description is terse, given that this entire volume is devoted to Lesson Study.

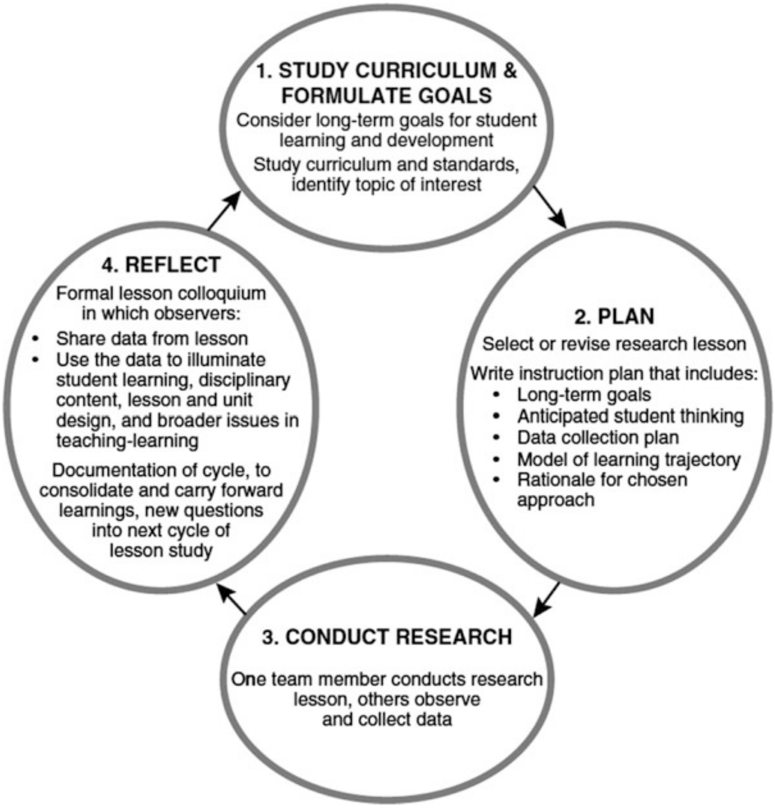

Fig. 1 Lesson Study cycle (From Lewis et al. 2006, p. 4)

The theory of action underlying LS is that such repeated inquiry cycles support the TLC's sense of community and, in an organic way, deepen and modify teachers' beliefs and understandings. This leads to changes in their pedagogical practices and deeper student learning.

1.3 An Introduction to the Teaching for Robust Understanding Framework and PD Structure

The TRU framework provides a theory of powerful learning environments (Schoenfeld 2015). Tools exist for supporting classroom practices in all disciplines (see, e.g., Baldinger et al. 2016, Schoenfeld and the Teaching for Robust Understanding Project 2016). Here we focus on the mathematics version of TRU, TRUmath.

The Five Dimensions of Powerful Mathematics Classrooms				
The Mathematics	Cognitive Demand	Equitable Access to Mathematics	Agency, Ownership, and Identity	Formative Assessment
The extent to which classroom activity structures provide opportunities for students to become knowledgeable, flexible, and resourceful mathematical thinkers. Discussions are focused and coherent, providing opportunities to learn mathematical ideas, techniques, and perspectives, make connections, and develop productive mathematical habits of mind.	*The extent to which students have opportunities to grapple with and make sense of important mathematical ideas and their use. Students learn best when they are challenged in ways that provide room and support for growth, with task difficulty ranging from moderate to demanding. The level of challenge should be conducive to what has been called "productive struggle."*	*The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core mathematical content being addressed by the class. Classrooms in which a small number of students get most of the "air time" are not equitable, no matter how rich the content: all students need to be involved in meaningful ways.*	*The extent to which students are provided opportunities to "walk the walk and talk the talk" – to contribute to conversations about mathematical ideas, to build on others' ideas and have others build on theirs – in ways that contribute to their development of agency (the willingness to engage), their ownership over the content, and the development of positive identities as thinkers and learners.*	*The extent to which classroom activities elicit student thinking and subsequent interactions respond to those ideas, building on productive beginnings and addressing emerging misunderstandings. Powerful instruction "meets students where they are" and gives them opportunities to deepen their understandings.*

Fig. 2 The five dimensions of robust mathematics classrooms – the Teaching for Robust Understanding (TRU) framework for mathematics, TRUmath

1.3.1 The Teaching for Robust Understanding Framework

The five dimensions of TRUmath are given in Fig. 2.

As described in detail by Schoenfeld (2013, 2014, 2017), the derivation of the framework included a comprehensive literature search regarding factors affecting student outcomes and the distillation of that list into five categories framed so that each is "actionable" – each can be the focus of professional development. Examinations of videotapes of practice and the use of a scoring rubric (Schoenfeld et al. 2014) documented a positive relationship between scores assigned to classroom practices using the rubric and student performance on measures of mathematical proficiency. Students from classrooms that rate increasingly well along the five dimensions are increasingly knowledgeable and resourceful thinkers and problem solvers.

TRU has several features that afford teacher learning, some of which are enumerated below.

1. The TRU framework provides mechanisms for focusing on what matters in instruction. TRU highlights key aspects of practice for collaborative teacher inquiry, experimentation, and reflection.

Teachers and administrators in the USA have few opportunities to develop shared goals (Grossman and McDonald 2008). Consequently, they can struggle to find common ground when beginning to talk about important problems of practice. This,

Observe the lesson through a student's eyes	
The Mathematics	• What's the big idea in this lesson? • How does it connect to what I already know?
Cognitive Demand	• How long am I given to think, and to make sense of things? • What happens when I get stuck? • Am I invited to explain things, or just give answers?
Equitable Access to Mathematics	• Do I get to participate in meaningful mathematical learning? • Can I hide or be ignored?
Agency, Ownership, and Identity	• Do I get to explain, to present my ideas? Are they built on? • Am I recognized as being capable and able to contribute in meaningful ways?
Formative Assessment	• Do classroom discussions include my thinking? • Does instruction respond to my thinking and help me think more deeply?

Fig. 3 Observing a mathematics lesson from the student perspective

in turn, makes it harder for them to work together to build and achieve shared goals. Even when there is time for collaborative meetings, focus and coherence can be a challenge (Horn and Kane 2015).

The perspective and language offered by TRU serve as a mechanism for boundary-crossing (Akkerman and Bakker 2011; Star and Griesemer 1989) and provide a means of communication among teachers at multiple grade levels and administrators. The five dimensions of TRU support teachers in inquiring into and reflecting on each other's routine practice. The hard work of coming to understand what the TRU dimensions can look like in classrooms helps teachers, administrators, and other stakeholders build shared understandings and goals. This facilitates collective work on problems of practice.

2. In the cultural context of the USA, TRU involves a fundamental shift in perspective from focusing primarily on the activities and/or materials students engage with to a focus on the ways in which students experience mathematics, as supported by engagement with carefully designed activities and materials.

Focusing on the ways students experience instruction is a fundamental point of alignment between TRU and LS. A key question is, "What does math class feel like, from the point of view of the student?" This perspective is represented in Fig. 3, drawn from the TRU Observation Guide (Schoenfeld and the Teaching for Robust Understanding Project 2016).

It can be challenging for teams of teachers in the US to move their conversations beyond scheduling activities, organizing curriculum, and naming topics to be taught. Such conversations are necessary, but they rarely reach the conceptual depth that leads to powerful teacher learning (Horn et al. 2017). TRU shifts conversations away from such activities and materials to what students *perceive and experience*, supporting teachers in exploring goals and orientations toward teaching and learning. Questions such as "What does it feel like for a student to experience these mathematical ideas for the first time? How does this connect to what students already know? What's confusing, interesting, or useful about this?" do not have easy answers. They address what really matters for students and are worthy of extended, collaborative exploration.

3. There are many different ways to design mathematically rich learning environments. Accordingly, the TRU framework does not prescribe specific teaching methods. Professional development aligned with the TRU framework helps teachers problematize their instruction, building on their strengths and refining their practices with the goal of providing their students with increasingly rich experiences along each of the five TRU dimensions.

The TRU framework and TRU tools open up instruction for inquiry and reflection while respecting teacher professionalism and autonomy. Specifically, asking "How are students experiencing the mathematics along this particular dimension? How can those experiences be made richer?" allows teachers the freedom to work within their own styles while building shared understandings of and commitment to a set of goals known to help students become powerful mathematical thinkers. TRU is not laissez-faire: reflecting using the TRU dimensions in a particular context may well reveal that one particular approach in that context is more likely to foster deep student engagement with the content, or provide more equitable access to it, than another approach. But TRU does not tell teachers what to do. It supports and enhances reflection on aspects of instruction that matter.

1.3.2 TRU-Based Professional Development

TRU-based professional development (TRU-PD) leverages the affordances of the TRU framework described above – explicit foci, shift to the student perspective, and the structured problematizing of teaching methods – to help teachers build communities of inquiry around important problems of practice. TRU-PD entails a series of inquiry cycles in which teachers negotiate their visions of teaching and learning mathematics collaboratively, reflect on artifacts of practice (including video), and try out agreed-upon approaches with their students. Each stage of this inquiry cycle is supported by the explicit foci and boundary-crossing language built into the TRU framework.

Once a TLC has been introduced to TRU, it begins the inquiry process with collective reflections (top part of Fig. 4). The TRU framework facilitates the choice of a problem of practice[2] that will be central to their collaborative inquiry. Having

[2]Some examples of problems of practice and the TRU dimensions that support them are, "How can we support students to feel ownership over the mathematics and discuss each other's mathematical ideas and strategies?" (TRU Dimension 4: Agency, Ownership, and Identity) or "How can we support students to struggle productively?" (TRU Dimension 2: Cognitive Demand).

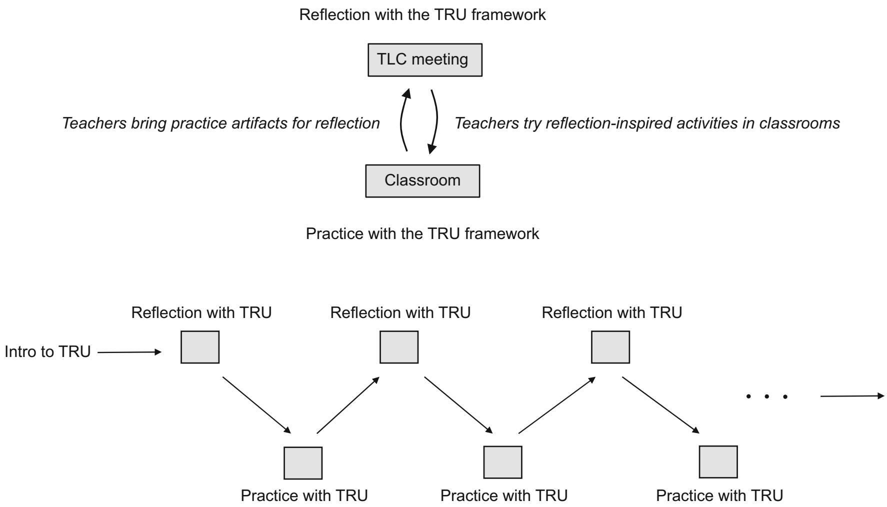

Fig. 4 The TRU inquiry cycle

chosen a focal problem of practice, the teachers identify relevant activities or techniques to try in their classrooms. The framework orients them to what they might try, but they are free to experiment with a variety of methods.

At this point, their inquiry shifts into their classroom practice. Teachers enact their interventions, collecting artifacts that will ground their subsequent collaborative reflections. Those artifacts might include replays or otherwise rich descriptions of what took place (Horn 2005), student work (Kazemi and Franke 2004), or video (Sherin et al. 2009).

In the next TLC meeting, the teachers report and reflect on what happened, using tools such as the *TRU Conversation Guide* (Baldinger et al. 2016) and the *TRU Observation Guide* (Schoenfeld and the Teaching for Robust Understanding Project 2016) to focus their reflections on students' experiences[3]. They then plan another intervention to refine their understanding of their chosen problem of practice, and the cycle again returns to the classroom.

Through the repeated cycles of reflection and practice shown in Fig. 4, TRU-based PD aims to develop teachers' capacity to collaborate on enriching students' experiences of mathematics and mathematics instruction by building routines of inquiring into everyday practice – an explicit goal that extends beyond typical Lesson Study. Working collaboratively with multiple TRU inquiry cycles is aimed at building a sense of teacher community and supporting structured collective reflection on teachers' attempts to create increasingly rich mathematical learning environments.

[3]See http://truframework.org for some of the tools developed to support teachers' planning, observation, and reflection.

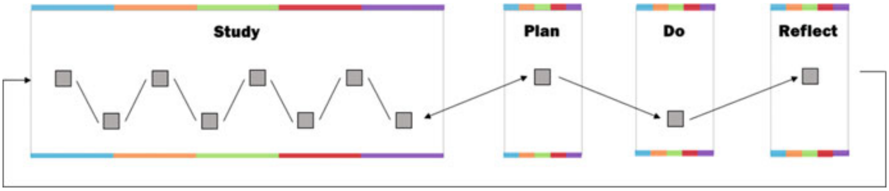

Fig. 5 TRU-LS: A synthesis of TRU and Lesson Study

1.4 *The Synthesis of TRU and LS*

The goal of synthesizing TRU and LS is to leverage the strengths of both TRU-PD and LS, employing TRU-PD activities and the TRU framework to address the challenges of shifting LS from the Japanese to the US context. Research has highlighted a number of these challenges while concurrently enumerating critical features deemed necessary for Lesson Study to be effective (Fernandez et al. 2003; Perry and Lewis 2009; Takahashi and McDougal 2016; Watanabe et al. 2008). One such critical feature is that a significant amount of time needs to be spent on *kyouzai kenkyuu*[4], literally translated as the "study of materials for teaching." Practically, however, as Wang-Iverson and Yoshida (2005) describe, *kyouzai kenkyuu* focuses not just on teaching materials but also includes investigation of "students' prior knowledge, learning experiences, state of learning, and understanding, which makes it possible for teachers to be able to anticipate students' reactions and solutions" (p. 152). As discussed below, this feature is addressed with TRU's integration into LS and its focus on building expertise about what matters in mathematics teaching and learning. Specifically, TRU-LS does so by providing teachers with opportunities for in-depth explorations of student thinking in their own classrooms, while laying the groundwork for and implementing Lesson Study research cycles occurring on a larger time scale. Two main TRU-based modifications of Lesson Study produce the synthesis and tackle these challenges.

As shown in Fig. 5, TRU-LS incorporates the TRU framework into each stage of the Lesson Study process. TLCs engaged in TRU-LS use the TRU framework to frame each stage of the Lesson Study process. TRU helps teachers select, study, and refine their research theme and theory of action and provides explicit focus for *kyouzai kenkyuu*. TRU observation tools help teachers focus their data collection during the research lesson on what students experience. The TRU framework also guides the final commentary during the post-lesson discussion. In the US context, where students' potential experiences of mathematics are not always central to

[4]Focused attention on the mathematics and to student thinking is essential in the US context. Curricula in the USA vary widely and are very uneven in quality, so teachers in the USA cannot depend on the affordances of curricula that are available in Japan. Moreover, teachers in the USA have not, as a rule, had nearly as much opportunity as Japanese teachers to reflect on the impact of classroom materials and practices on student thinking.

conversations about teaching, the focus that the TRU framework provides is essential. It helps keep the focus on building expertise around student thinking. More generally, the use of TRU in this way makes explicit many of the underlying values and practices that can be tacit in traditional Japanese Lesson Study and are “carried” by cultural tradition (Takahashi 2015).

In Japan, the teaching community is steeped in Lesson Study, so teachers who have not been part of the research lesson planning team can still learn a great deal from the framing questions, enactment of the lesson, and the commentaries. That is not necessarily the case in the USA. Accordingly, the TRU-LS process insures that all members of the department or TLC engage with all aspects of the research lesson before it is taught. As seen in Fig. 5, the TRU inquiry cycles, in which all teachers participate, form the backbone of the study phase of Lesson Study. The research theme and preliminary theory of action for the research lesson are developed in the TLC. All TLC members, including the lesson planning team, engage with the research theme and theory of action through repeated cycles of reflection and practice; relevant literature is referenced, although (given the realities of US teaching) typically not with the intensity and thoroughness of classical *kyouzai kenkyuu*. This ongoing work allows for collective refinement of the research theme and theory of action. Later, as the planning team works to design the research lesson, it brings the core mathematics of the lesson to the larger community for discussion. That conversation includes discussions of the mathematics itself, what the students are likely to know, anticipated student responses, and how mathematical ideas develop throughout the lesson as well as specific design elements – an analog of the work in selecting and analyzing the *hatsumon* in traditional LS. These collective conversations inform the research lesson planning and also raise possibilities for intervention and reflection that the whole department can try out in ongoing inquiry cycles. As shown in Fig. 5, the plan, do, and reflect phases of Lesson Study (cf. Figure 1) interact in ongoing ways with collective study by the TLC.

In TRU-LS, TRU inquiry cycles allow the TLC to leverage the LS process for community-wide development and learning. The TRU cycles also support community building and the collective negotiation of teachers’ visions of teaching and learning mathematics, which is essential for effective LS cycles. Finally, they help the learning done in a LS cycle permeate teachers’ regular classroom practice.

The changes to Lesson Study described here – engaging the TLC in small inquiry cycles within the larger inquiry cycle that is LS and using the TRU framework to scaffold conversations at each stage of the Lesson Study process – are aimed at bridging the divide between LS as designed for the Japanese context and the reality of teachers’ working lives in the USA. They offer the potential to enhance both TRU-PD and LS. To TRU-based PD the changes add the benefit of involvement in larger-scale and more intense design than conducted in TRU inquiry cycles, supporting teachers in delving more deeply into content and student thinking. To LS, the changes add aspects of community building necessary in the USA and the kinds of explicitness and focus on key aspects of instruction that increase the likelihood both of successful lesson design and of collective learning from participating and reflecting on that design.

2 Early Signs of Impact

This section summarizes instances of practice from three school sites to show how early implementations of TRU-LS are taking shape. We selected these examples to highlight consequential differences between TRU-LS and typical LS implementation in the USA.

For the past 3 years, the TRU-LS team has collaborated with high school mathematics departments in the Oakland (California) Unified School District (OUSD) in designing and implementing TRU-LS. OUSD high schools vary substantially in terms of size, schedule, personnel, and priorities; even though there is a common curriculum in the district, implementation of that curriculum varies significantly across sites. Given that TRU-LS is designed to be responsive to local context, the examples demonstrate substantial variation in TRU-LS implementation, even though the implementations are consistent with the underlying design principles of TRU-LS.[5] That said, there is enough curricular consistency to produce overlap in research themes and research lessons.

The study phase of LS is the most extensive and a challenge to implement in the USA – teachers in the USA are not habituated to the deep study of curricula or student thinking and tend to move rapidly to selecting and/or designing tasks for implementation in research lessons. TRU-LS, while still proceeding more rapidly than is typical in Japan, tries to open things up by having a TLC focus on developing a research theme and a supporting theory of action. Here we discuss three examples from the study phase. We also present two examples from the combined plan, do, and reflect stages.

2.1 Establishing a Research Theme and Theory of Action with TRU-LS at Sites A and B

TRU-LS integrates TRU-based inquiry cycles with the study phase of Lesson Study with the goal of helping TLCs create and refine research themes with explicit underlying theories of action. Here, examples from sites A and B illustrate the selection and refinement of research themes resulting from focused attention on students' mathematical experiences. These examples also show how TRU-LS supported teachers' agency and helped them begin to develop routines of collaborating as a department about problems of practice, building the culture essential to LS.

[5] This is an essential, given our wish to respect teacher autonomy and help build local teacher communities.

2.1.1 Site A: Crafting an Initial Research Theme Through Reflection with the TRU Framework

The process began at site A with each teacher sharing one goal they set for the school year. Because this site had studied TRU previously, many of the teachers' goals were already framed by TRU. As teachers talked, it became apparent that three shared themes were emerging – supporting student explanations/arguments, building student ownership of their learning, and finding the "sweet spot" for productive struggle. To narrow the theme, the teachers discussed strengths and areas of growth for their current students. The language of TRU helped keep the discussion focused on the students as learners. Teachers voted on the theme they preferred, settling on the "sweet spot" of productive struggle. Teachers said they saw this idea as connected not only to Cognitive Demand but also to Equitable Access; material has to be "within reach" if all students are to be productively engaged. At the end of the first meeting, teachers agreed to try strategies aimed at supporting productive struggle in their classrooms prior to the next full department PD meeting. (This highlights the use of TRU inquiry cycles as part of the development of the LS research theme.)

With the goal of deepening their collective understanding of productive struggle and deciding which aspects of productive struggle they wanted to focus on in their research lesson, the TLC began its second meeting by watching and reflecting on a video of students in an Algebra 2 class grappling with a challenging problem. The teachers organized their observations using a TRU Cognitive Demand observation tool. They recorded what student behaviors they saw as productive struggle, what factors constrained productive struggle, and what strengths each student brought to the task.

Teachers noted that they saw students in the video trying an alternate strategy when their current strategy was not working as they had expected and asking each other whether a solution or process "makes sense" – productive behaviors that they wanted to support. Then, a description of two students who put their heads down during the task as "disengaged" prompted a discussion of how one might support students to engage productively if their "immediate response" to a math problem is "I don't get it, I'm done." This, in turn, led to discussions of what kinds of classroom environments support productive engagement – students won't persevere and engage in productive struggle if they don't feel safe enough to take risks. These observations led to an enhanced research theme proposal: "How do we set up students so that they are comfortable engaging in productive struggle and how do we arrange what they are grappling with so that they can make meaningful progress toward the learning target and standards?"

This narrative illustrates how collaborative reflection on a classroom video, framed by the shared language of the TRU framework, supported selection and refinement of a research theme during the study phase of TRU-LS. At the initial stages of the TRU-LS study phase, the TRU framework helped the TLC articulate and focus on meaningful problems of practice. Joint reflection on classroom video allowed teachers to deepen their noticing of student experiences. This, in turn, supported refining and negotiating their collective understanding of the research theme. [We note that in Japan, a focus on the students' experience of the mathematics can be taken as a given. That is not necessarily the case in the USA. The TRU framework oriented teachers toward the student experience, and the video provided a grounding for the conversations that kept them from becoming too abstract.]

2.1.2 Site B: Refining a Research Theme and Developing a Theory of Action Through TRU Inquiry Cycles

Similarly, the TLC at site B revised its research theme and developed a theory of action by engaging in TRU-based inquiry cycles of reflection and practice. These cycles helped them develop and sustain the three lenses of *kyouzai kenkyuu* that Fernandez et al. (2003) observed that their Japanese colleagues deem critical – the researcher lens, the curricular lens, and the student lens. Here we describe a department meeting several months into the TRU-LS process in which teachers reflected on artifacts of practice generated when they tried an intervention developed to help them learn about their draft research theme. The TRU framework helped teachers reflect on the artifacts they brought to the meeting and translate their observations into goals for teaching and learning. These resulted in a revised research theme and theory of action.

As had been the case at site A, teachers at site B developed their research theme through reflections on their goals, students' strengths and challenges, and the TRU framework. Their initial research theme was, "Building student perseverance/capacity to struggle productively, together." They connected this theme to the Cognitive Demand and Agency, Ownership, and Identity dimensions of the TRU framework. (Students are likely to persevere only if they have a sense of agency, and such agency is built by having struggled productively – which means that tasks are cognitively demanding but within reach.)

To engage in the practice part of the TRU inquiry cycle, teachers chose to explore a teaching strategy that they believed would help them see their research theme of student perseverance "come to life."[6] They chose a teaching strategy called Three Things: When students are stuck, the teacher would ask them to state three things they know about the problem and three things they are wondering about. Teachers conjectured that this strategy would help students think about what they already know when they got stuck and that it might help students become more independent. They hoped that it would prompt students to explain their thinking and that students might begin using it with each other.

[6]Note that TRU, with its emphasis on the student experience of the content, often leads teachers to consider pedagogical choices as well as content choices – the question being, "how do we enrich the students' experience of the content?"

In subsequent meetings, reflections on Three Things and discussions of classroom artifacts related to the theme of perseverance and productive struggle helped the community take up a student perspective on the research theme. Observations grounded in the shared language of TRU supported the teachers in building on one another's thinking about student perseverance and their efforts to support it. The discussion peaked during their fourth department meeting, slightly more than a month before their first research lesson.

At the beginning of this fourth meeting, the teachers discussed what happened in their classrooms when they tried Three Things. One teacher began the discussion by connecting her reflection to the TRU "equitable access" dimension. This prompted another teacher to reflect that in his experience, Three Things prompted more students to be more open about sharing their ideas verbally. Another teacher said that she had noticed that while Three Things encouraged students who were "borderline engaged" to talk more, it did not seem to support students who were more physically and audibly disengaged from classroom work. These observations stimulated a community discussion of what is meant by equitable access and gave rise to the question: "Does Three Things help us achieve *equitable* access to collaborative productive struggle?" A fourth teacher challenged whether Three Things was actually promoting collaborative productive struggle since the teacher, rather than the students, had provided the strategy.

This challenge led the teachers to examine more deeply each other's definitions of "persistence," a word they had used interchangeably with perseverance and productive struggle – but this time with attention to how persistence manifests for students who experience the challenge of mathematics problems differently. The facilitator encouraged them to try to disentangle their different definitions of persistence using the TRU dimensions. In the discussion that followed, teachers identified specific reasons that students might not persevere, tying their observations to TRU dimensions:

[Students might say] "Why are we doing this? This is so hard" (Cognitive Demand).
"Kids don't feel like they have a toolbox" (Equitable Access).
Fixed mindset – "I'm not good at math" (Agency/Ownership/Identity).

One teacher suggested that when teachers ask students to share three things when they are stuck, students are prevented from drawing on their "toolbox" themselves. From her perspective, students who failed to persevere because they felt as though they did not have the tools to move forward were not supported to persevere by teachers providing those tools for them; those students should be supported to find the tools they need themselves.

This comment led the teachers to wonder how and to what Three Things gives students access. While the strategy might give students more equitable access to the *content* by helping them continue to solve a problem and allowing them to hear strategies developed by other students, it might not give them access to mathematical practices, including perseverance, if they depended on the teacher to move them forward when stuck. If perseverance means individual and collective struggle without teacher intervention, having a teacher ask for three things does not help students develop that capacity.

This articulation of what the teachers really meant by perseverance, tied to their experiences with Three Things and the TRU dimensions, marked a significant step forward in developing a rich research theme tied to student experiences. By the end of this fourth department meeting, the teachers knew that while they cared about supporting perseverance so that students would have equitable access to content, they also cared about supporting perseverance so that students could draw on each other, rather than their teachers, for support during times of need. Moving forward, the teachers oriented toward finding ways to support students to draw on resources such as their peers, notes, or classroom artifacts with less teacher scaffolding.

This is precisely the kind of *problematizing* – inquiring into the whys and wherefores of one's teaching – that TRU is explicitly designed to support. Teachers' experiences at sites A and B demonstrate how TLC-wide TRU-based inquiry cycles, nested within the study phase of a Lesson Study cycle, supported teachers in developing a deeper research theme that was more directly tied to student experiences. The teachers developed a culture of collaborative inquiry into teaching and learning, testing, and discussing conjectures about teaching strategies, in advance of their work crafting and reflecting on research lessons.

2.2 Plan and Study: Interconnectedness Between the Planning Team and an Entire Department's Study Process

One significant difference between TRU-LS and traditional LS in Japan is the interconnection of whole department PD and the research lesson planning team. In TRU-LS, responsibility for planning the research lesson is assigned to a planning team – but that planning team interacts with the whole department in ongoing ways as it works on critical planning issues. Through this interconnection, *kyouzai kenkyuu* is further developed, particularly with regard to the collective development of the curricular and student lenses.

2.2.1 Site A: Refining a Lesson Plan Through a TLC Discussion Grounded in TRU

The TLC at site A, which was focusing on productive struggle (cf. Sect. 2.1.1), decided that its content focus would be on systems of linear equations. The planning team reviewed the relevant progression of Common Core standards from grade 7 through high school, examining tasks they believed would afford rich explorations of the content and also provide students with opportunities to engage in mathematical practices related to productive struggle.

The teacher who had volunteered to teach the research lesson said that his approach to the topic the previous year had been procedural and that he hoped to increase Cognitive Demand this year. Accordingly, the planning team began by working through a sample assignment from the previous year. This task offered possibilities for students to use multiple representations but in a largely procedural way.

The planning team then studied additional tasks that might open up the mathematics and offer additional challenges – but, they were hampered by a narrow interpretation of the Common Core content standards. They read the standards to say that students only needed to learn the slope-intercept form of linear equations and that they only needed to learn to solve pairs of simultaneous linear equations using substitution. This reading constrained their considerations concerning Cognitive Demand. For example, a task they examined was helpful in opening up the space of possible student strategies, but it was rejected from consideration because the equations in the task were presented in standard form.

The content-related question driving the planning team was: "What knowledge do we want students to leave this unit with?" Ultimately, the team decided that it wanted students to understand (1) that the solution set of a pair of linear equations is the set of points where the graphs of the two equations intersect and (2) the conditions under which such a system will have zero, one, or infinitely many solutions. The Formative Assessment Lesson "Classifying Solutions to Systems of Equations" – the lesson can be downloaded from http://map.mathshell.org/lessons.php?unit=8220&collection=8 at no cost – seemed promising in this regard, despite its length (the lesson was designed for 2–3 days) and its use of equations in standard form. The planning team decided to bring the FAL to the TLC so that everyone could do the mathematics and aid the team in adjusting the tasks to hit the "sweet spot" of productive struggle.

Doing the mathematics together at the TLC meeting generated a fruitful discussion that shaped the planning team's subsequent design decisions. TLC members agreed that the level of Cognitive Demand was too high, encouraging the planning team to think more deeply about how to modify the task without "scaffolding away" the most important mathematical ideas. For example, when the planning team proposed changing all the equations in the lesson to slope-intercept form in an effort to make the lesson more accessible, an Algebra 1 teacher challenged, "if you alter all these equations to be in slope-intercept form, then aren't you giving up a lot of the Cognitive Demand of this particular task?" Engaging the whole TLC broadened the collection of possible student strategies for comparing two equations. It expanded the set of connections that might be made, highlighting places where students might engage deeply with the content. Finally, TLC members also suggested changes to the tasks that could make it easier for students to access the central mathematics within the allotted time. The planning team followed up on these ideas in subsequent planning meetings.

This record of teacher conversations demonstrates ways in which coordination between TLC study meetings and planning team meetings, built into TRU-LS, enriches both the study and the planning phases of LS. The larger community's

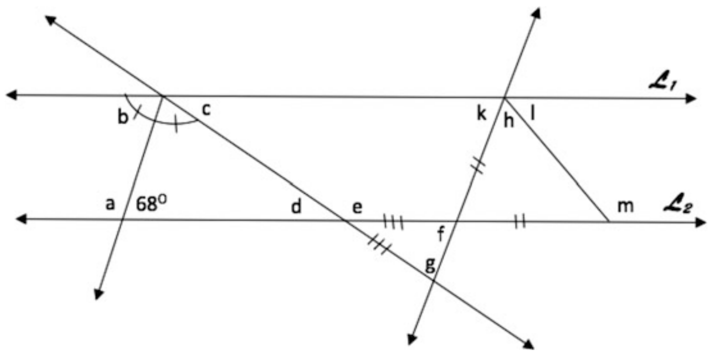

Fig. 6 The main task in the Group Angle Challenge lesson

knowledge of TRU and the Common Core enabled the planning team to think more deeply about the content of the research lesson than their consideration of the standards documents had afforded. At the same time, doing the mathematics together supported all of the teachers in the TLC in engaging more deeply with the content and issues of Cognitive Demand than if they had discussed these issues in the abstract and positioned them to better understand what would transpire in the research lesson. In this way, the synthesis of TRU and LS expanded the opportunities for all TLC members to learn about the mathematical content and practices through a student lens.

2.3 *Do and Reflect: Two Different Trajectories Through the Same Lesson Plan at Sites B and C*

Sites B and C independently chose to use the same OUSD lesson, the Group Angle Challenge, as the basis for one of their research lessons. The main mathematical activity in the lesson asks students to determine the values of labeled angles in Fig. 6, given that lines L_1 and L_2 are parallel.

The lesson begins with students working in groups to determine each angle and record an explanation of their reasoning in a table. Each group then makes a poster to record its findings. The lesson concludes with students from different groups visiting each other's posters to read and comment. The content focus of the lesson is rather limited, but the lesson does offer possibilities for student discourse and argumentation.

The planning teams at sites B and C chose this lesson for different reasons. Site B teachers hoped that the lesson would help students learn to turn increasingly to classroom resources and each other for help instead of to the teacher. Site C teachers hoped that the lesson would help them support students to justify their thinking and critique the mathematical thinking of others. Thus both sites focused on aspects of dimensions 2 and 4 of TRU, Cognitive Demand and Agency/Ownership/Identity.

Yet their specific research themes and theories of action led them to focus differently in observing and reflecting on their research lessons. Both sites reflected deeply on their students' experiences but very differently.

The examples that follow indicate how the nonprescriptive but focused character of the TRU framework can play out productively in practice. The Group Angle Challenge lesson can be modified in myriad ways, not all of which would support rich opportunities to learn in classroom discourse. Leaving things wide open could lead to unhappy results; but forcing teachers down any particular path – even if the direction taken is useful – runs the risk of engendering resistance and undermining attempts to build teacher community. That teachers at the two sites were able to pursue their specific interests was enfranchising, contributing to teacher agency and the ongoing development of teacher community. At the same time, TRU helped to problematize practice in focused and productive ways.

2.3.1 Site B: What Would Support Students to Turn to Each Other as Resources? A Focus on Agency, Ownership, and Identity

In post-research lesson reflections, site B teachers and their final commentator focused largely on the ways in which students sought help from each other and their teachers and the role teacher interventions played in student perseverance. They commented that few students had drawn on the clearly relevant posters that students had developed over the past week and that teachers expected students to use as resources. Although some students quickly referred to their notes or to posters, many did not do so – seemingly at a loss for how to proceed without heavy scaffolding from a teacher. These observations challenged the teachers' theory of action. The resources and questions that they had developed had not supported as many students to struggle productively together as they had hoped.

Site B teachers did not explicitly categorize their observations into the TRU dimensions, but the Cognitive Demand, Equitable Access, and Agency/Ownership/Identity (AOI) dimensions featured in their conversations.[7] The final commentator encouraged them to focus on AOI, especially with respect to the ways that teacher interventions can both enhance and undermine student agency during group work. She highlighted observations in which teachers had described having to scaffold student thinking more than they felt was appropriate for their goals.

She also discussed a specific observation that she felt had implications for student agency. She had heard a student discuss an idea with confidence and certainty when the teacher wasn't present. But when the teacher dropped by to check in, the student had asked a question about that idea as if she hadn't already thought of it. The teacher, unknowingly, went over some of the ideas the student had already reasoned

[7] Although organizing observations according to the TRU dimensions can be useful, it is certainly not necessary. Indeed, the goal is for the TRU language and perspective to become internalized, so that they serve as a natural framing for discussions.

through. When the teacher left, the student attempted to reproduce the teacher's phrasing and approach, seemingly losing track of the way she had dealt with it herself. This kind of interaction has the potential to undercut student agency.

These observations deepened the group's conversation about AOI. The teachers reflected on the implications of their interventions during group work, noting that high school students' mathematical agency might be fragile despite a teacher's best efforts. The final commentator also encouraged the teachers to think of ways that the resourceful mathematical work being done by some groups might be made public to the class, so that students could learn about using resources from each other rather than their teachers.

In subsequent TRU-LS cycles, site B teachers focused further on helping their students develop mathematical resources that they could draw on during group work. They also attended more carefully to the ways in which students were supported to engage with the big mathematical ideas in their lessons. They observed that students would be better positioned for collaborative productive struggle if the mathematical ideas present in a lesson were worth the struggle and if available resources engaged students with those ideas.

2.3.2 Site C: Reflection with the TRU Framework Supports Shifts in Theory of Action in a Subsequent TRU-LS Cycle

In their post-research lesson discussion, site C teachers noted that although students had justified their reasoning to each other while working in small groups on the Group Angle Challenge, much of that mathematical talk did not make it onto their posters or in comments to their peers. This observation ran against their theory of action. The teacher leading the research lesson had modeled "what critiquing reasoning looks like" and the lesson format provided students opportunities to critique each other's work. However, students' critiques contained neither the mathematical thinking the planning team had expected nor the mathematical thinking that had been observed in the small group work that preceded the critiques.

The teachers recorded these observations in the context of the Mathematics and Agency/Ownership/Identity dimensions. Under Mathematics they wrote, "Students knew how to justify verbally, writing didn't match." They also suggested "Maybe tell them what kind of info to include in the justifications," thinking that students' mathematical writing might be richer if the teachers were explicit about the criteria they had in mind for richness. Under AOI they wrote "Student critiques were vague," referencing the AOI dimension's attention to students' critiquing and building on each other's thinking. They added "Students did a good job of holding each other accountable while solving," but "This thinking didn't make it onto the poster."

The final commentator started by touching briefly on each of the five TRU dimensions, highlighting some of the observations that had been made by the teachers. She then turned to formative assessment (TRU dimension 5), which had not received much attention during the teachers' lesson planning or reflection. After discussing the lesson through the lens of formative assessment, she turned to next steps, suggesting that the team revisit their theory of action in light of some of the teachers' planning notes.[8] She drew the teachers' attention to where they had written, "Our goal is for students to be able to (1) think (2) conjecture (3) critique and (4) revise... 'Revise' allows students to know that it's okay to make mindful mistakes." She prompted the teachers to think about how, if at all, this final step of "Revise" showed up in their research lesson. She drew from the TRU Observation Guide, highlighting two Student look-fors ("Provides specific and accurate feedback to fellow students" and "Makes use of feedback in revising work") and one Teacher look-for ("Flexibly adjust content and process, providing students opportunities for re-engagement and revision"). She suggested that if the students had been given the chance to revise their work, they would have better understood the purpose of giving precise, constructive feedback.

As they moved into their second Lesson Study cycle, teachers at site C decided to keep the research theme from their first LS cycle, "building students' mathematical reasoning through student justification." However, reflections on first research lesson led to significant changes in their theory of action. The teachers had selected the Group Angle Challenge for their first research lesson because they felt it would provide students opportunities to justify their answers. They were now more aware of the limitations of the original curricular task: because there was only one correct answer for each angle measure and only one or two ways to explain each answer, students' justifications were not as central to the task as they could have been. One teacher suggested that "more rigorous open-ended tasks" would offer more opportunities for students to share a variety of justifications and critiques.

As a result of these reflections, teachers adjusted their theory of action for their second LS cycle to incorporate the four-step justification process highlighted in the final commentary. They chose a richer and more challenging set of tasks for the core content of the lesson, and they used a TRU Formative Assessment lens when considering instructional strategies, students' learning experiences, and task design in preparation for their second research lesson.

This sequence of events illustrates how TRU and Lesson Study work in synergy. TRU helped frame teacher reflection, making it more productive and powerful. Furthermore, the use of an external commentator versed in TRU broadened and deepened the set of focal issues the TLC took into account.

[8] Planning for the research lesson was supported by and recorded in a TRU-Lesson Study Discussion Guide, which yields a lesson plan similar in detail to those developed in traditional Lesson Study.

3 Discussion

To take hold and flourish, any mechanism for teachers' professional growth must mesh with the context in which it is implemented. In Japan, Lesson Study is deeply embedded in supportive school contexts and structures. Teachers are provided time and space intended explicitly for collaboration; they are supported in working together; there is a decades-long tradition of attending to content and practices in instruction and professional development; and curricula provide a rich and stable base for curricular inquiry. Under such circumstances, Lesson Study can function as intended, supporting the continuous development of Japanese teachers' knowledge and practices.

The US context for professional development differs substantially. Although conditions vary, teachers in the USA typically have few opportunities for collaboration. Efforts to support professional growth and collaboration are typically undertheorized, if they are theoretically grounded at all. Teaching practice is largely considered private; when external observers walk into a classroom, the purpose is often evaluative. Curricula vary substantially in the affordances they provide both for content and practices, and there is much less of a tradition of attending to student thinking than in Japan. TRU-based professional development was designed in and for the US context. Explicit attention is given to community building among teachers. TRU offers a structure and tools that help teachers focus on key aspects of instruction while not being prescriptive in ways that would undermine teacher agency.

A primary affordance of Lesson Study is the learning that comes from teachers' extended study of important mathematical ideas and the ways that students make sense of them. A primary affordance of TRU is the explicitness with which it highlights productive aspects of the learning environment, with a focus on the students' experience of the mathematics. The five dimensions of the TRU framework provide a clear framing that highlights essential aspects of classroom practice. This approach has been useful across the USA, providing grounding for professional development efforts that might otherwise be scattershot (Schoenfeld et al. press). In addition, the TRU framework helps to make explicit the goals and theories of action for Lesson Study research lessons and observations of them – something essential in the USA but also helpful in Japan, where key aspects of practice are largely tacit and passed on by tradition. The use of TRU inquiry cycles, which bring aspects of research lesson design to an entire teacher learning community, helps to prepare all members of that TLC to profit from observing the research lesson and hearing the lesson commentaries. It also supports the development of teacher community. In these ways, TRU-LS provides a mechanism for the functional enrichment and adaptation of LS to the US context.

TRU-Lesson Study has unique potential for broad implementation across the USA and other nations with similar cultures of teaching. As this volume demonstrates, Lesson Study is a powerful mechanism for professional development, when the culture supports it. The TRU part of TRU-LS, which is powerful on its own – see

Schoenfeld et al. (press) for evidence of its impact – helps to establish fertile ground in which the Lesson Study part of TRU-LS can take hold and flourish. Admittedly the TRU-LS work is in its early stages, and the evidence presented in Sect. 2 is preliminary. This is as it must be: it takes time to build supportive contexts, and the development of teacher knowledge and expertise is, like all expertise, a process that takes thousands of hours of concerted practice and reflection. However, the evidence thus far suggests that there is some reason for optimism.

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